VOLUME 86 NO. HY7
JULY 1960

JOURNAL of the

Hydraulics Division

PROCEEDINGS OF THE

AMERICAN SOCIETY

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The index for 1959 was published as ASCE Publication 1960-10 (list price $2.00); indexes for previous years are also available.

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Journal of the

HYDRAULICS DIVISION

Proceedings of the American Society of Civil Engineers

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Papers

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DRAG FORCES IN VELOCITY GRADIENT FLOW*

By Frank D. Masch, A. M. ASCE[1] and Walter L. Moore, M. ASCE[2]

SYNOPSIS

An exploratory investigation was made of the drag coefficient for a circular cylinder as influenced by a velocity gradient along its axis. Some of the three-dimensional aspects of the flow mechanics were described qualitatively. Experiments indicated that the local drag coefficient varied significantly along the length of the cylinder. For velocity gradients in the range of engineering significance, local drag coefficients may be affected by as much as 40%.

INTRODUCTION

The evaluation of the hydrodynamic forces on a solid body in a flowing fluid is of primary importance in many engineering problems. In a few simple cases, these forces can be evaluated by analytical methods alone. In most practical problems, it is necessary to resort to a combination of analysis and experiment. An experimentally determined drag coefficient is commonly used in estimating drag forces.

Experimental determinations of drag coefficients for various shapes usually have been made in a stream with a nearly constant velocity. However, many problems of engineering importance arise in which the velocity of the fluid

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* Presented at a meeting of the Hydraulics Division, ASCE, Los Angeles, Calif., February 9, 1959.

[1] Formerly, Instr. in Civ. Engrg., Univ. of Texas; now, Science-Faculty Fellow, Div. of Hydraulic and Sanitary Engrg., Univ. of California.

[2] Prof. of Civ. Engrg., Univ. of Texas, Austin, Texas.

stream is not constant. For example, velocity variations will be present if the fluid movement is the result of wave motion, or if the fluid motion is retarded by the resistance along a boundary. More specific problems, in which a velocity gradient may be significant, are; wave forces on piers or other structures, wind forces on buildings or stacks, resistance due to piers in streams and channels, forces on sediment particles, and drag forces on surfaces irregularities in a boundary layer.

Comparatively little effort has been devoted to the study of drag forces in a velocity gradient. S. F. Hoerner, (1)[3] has treated two aspects of the problem. He summarizes the work of Tillman, Wiegard, and others, on the drag of protuberances in a turbulent boundary layer. For these surface irregularities, whose height is less than the boundary layer thickness and whose width is large in relation to their height, the indications are that a satisfactory estimate of the drag force can be made assuming the drag coefficient to be the same as for flow with no velocity gradient. An effective drag coefficient based on the velocity at the top of the protuberance can be computed by simply summing the forces at successive layers accounting only for the variation of the velocity. That is, at each layer, the drag coefficient is considered to be the same as for two-dimensional flow. Hoerner also presents approximate data on the velocity profiles near the ground for wind and on drag coefficients for several building shapes.

A number of papers are available, dealing with the motion of solid bodies in rotating fluids. H. B. Squire's (2) has dealt with several aspects of this problem and refers to a group of papers on this subject presented in the period from 1915-25.

W. R. Hawthorne (6) made a theoretical and experimental study of the flow about struts and airfoils in a flow with a spanwise velocity gradient. His analysis of vortex development is of special interest, but as he points out it is not applicable to blunt-nosed bodies. He assumes that the induced spanwise velocity components are small and he does not account for any effect in the wake. For these reasons his analysis, although based on similar concepts of the mechanics of the flow, is not applicable to the present investigation.

FORMULATION OF PROBLEM

The variables normally considered in drag force studies can be grouped into a dimensionless equation of the form

$$C_D = \frac{F_D}{A \rho V^2/2} = \phi \ (\mathbf{R}, \text{form}) \qquad (1)$$

in which C_D is the drag coefficient, F_D the drag force, A the cross-sectional area of the object taken perpendicular to the flow, ρ the mass density of the fluid, V the velocity of the approaching fluid, and $\mathbf{R}$ the Reynolds Number.

This equation represents the average drag coefficient as a function of the Reynolds Number and the shape or form of the object under consideration. The application of this equation to two-dimensional flow problems normally assumes a uniform velocity distribution approaching the object in question.

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[3] Numerals in parentheses, thus (1), refer to corresponding items in the Bibliography.

This investigation involved the introduction of additional variables in Eq. 1 to permit an evaluation of the drag forces on a circular cylinder with its axis normal to the approaching flow as affected by a linear velocity gradient in the direction of its axis.

A dimensional analysis enables a velocity gradient parameter to be formulated into the drag coefficient equation as

$$C_D = \frac{F_D}{A\,\rho\,V^2/2} = \phi'\left(\mathbf{R},\ \text{form},\ \frac{\Delta V\ D}{\Delta Y\ V},\ \frac{D}{Y_o}\right) \ldots\ldots\ldots\ldots (2)$$

The geometrical aspects are represented by the form and the ratios of cylinder diameter to cylinder length, D/Y_o. If the form is restricted to a circular cylinder and the ratio, D/Y_o, is held essentially constant, these two terms will not affect the drag coefficient.

The velocity gradient parameter, $\frac{\Delta V\ D}{\Delta Y\ V}$, is a dimensionless combination of variables in which $\Delta V/\Delta Y$ is the slope of the velocity profile and $\frac{D}{V}$ is the ratio of the diameter of the cylinder to the velocity measured at the midpoint of the cylinder. If the velocity distribution is uniform, the slope, $\Delta V/\Delta Y$ is zero, thus causing $\frac{\Delta V\ D}{\Delta Y\ V}$ to vanish leaving the drag coefficient a function of the Reynolds Number only.

The Reynolds Number may be formed from the velocity at the mid-point, as determined from the velocity profile measurements, the diameter of the cylinder, and the kinematic viscosity of the fluid. For a range of Reynolds numbers between 10,000 and 100,000, there is practically no variation of the average drag coefficient with the Reynolds Number. In this range of Reynolds numbers, the shear forces on the cylinder are very small in relation to the pressure forces, and the total drag can be considered as being primarily form drag.

Under the limitations previously described, the form, D/Y_o, and $\mathbf{R}$ may be eliminated from the functional relationship of Eq. 2, leaving the drag coefficient a function of the velocity gradient parameter only.

As an aid in analyzing the mechanics of this problem, a local drag coefficient was formulated similar to the average drag coefficient, that is,

$$c_D = \frac{\dfrac{f}{\Delta Y\ D}}{\rho\left(\dfrac{v^2}{2}\right)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which f is the local drag force, $\Delta Y\ D$ is the projected area of an element of the cylinder, and v is the local velocity of the fluid approaching the element. The local force, f , may be determined from an integration of the pressure distribution around the cylindrical element, or

$$f = \int_0^{2\pi} p\ r \cos\theta\ d\theta \ldots\ldots\ldots\ldots\ldots (4)$$

in which p is the pressure intensity at any p t., r the cylinder radius, and θ the angular position on the cylinder measured from the upstream element of the cylinder.

Combination of this local force with the corresponding measured local velocity will give the local drag coefficient. This computation, repeated at several

elements along the longitudinal axis of the cylinder, will give the variation of the local drag coefficient along the cylinder.

A consideration of the mechanics of flow around the cylinder will indicate that the velocity gradient will induce flow components along the axis of the cylinder. Along the upstream element of the cylinder (the stagnation line), the stagnation pressure will be greater at the end of the cylinder, where the velocity is high, than at the end where the velocity is low. This will produce a pressure gradient along the axis of the cylinder and induce a flow along the stagnation line toward the low-velocity end of the cylinder. In a similar way, the reduced pressure in the wake will be affected by the local approach velocity. At the high-velocity end of the cylinder, the pressure in the wake will be less than at the low-velocity end, thus, inducing a longitudinal flow toward the high-velocity end of the cylinder, in the wake zone.

The effect of these longitudinal flows on the local drag coefficient at various locations along the cylinder may be anticipated as follows. At the high-velocity end of the cylinder the longitudinal flow along the stagnation line would be diverting fluid away and, thus, reducing the pressure intensity on the upstream side of the cylinder. At the same time, the longitudinal flow in the wake zone would be forcing fluid toward the high-velocity end and thereby raising the pressure intensities on the downstream side of the cylinder. Both of these effects would act to reduce the drag force on the cylinder and, hence, to reduce the local drag coefficient at the high-velocity end of the cylinder. It might be anticipated that the opposite effect would be true at the low-velocity end of the cylinder, and the local drag coefficient would be high. However, it is not necessarily true that this value will exceed the one for normal two-dimensional flow.

This rather crude description of the secondary flows may help to explain some of the essential features of the flow mechanics, but it appears that a more adequate description can be developed in terms of vortex generation. Some concepts bearing on this approach are described by Hawthorne (6) and by Squire and Winter (10) but more study is needed to develop them for application to the problem of this investigation.

EXPERIMENTAL INVESTIGATION

Experiments were made to explore the variation of the local drag coefficient along a cylinder. A 1-in. diam hard copper tube was mounted vertically at the center of a 12-in. wide glass-walled flume in which a flow of water was maintained at a depth of 0.8 ft. The flow rate was adjusted to give Reynolds Numbers at mid-depth in the range between 18,500 and 22,500, a region where the normal two-dimensional drag coefficient is constant. With the Reynolds Number in this range, and the form and D/Y_o held constant, these factors were eliminated as a source of variation in the measured drag coefficients. Fig. 1 is a schematic diagram of the test arrangement.

Since the flow had a free surface it might appear that the drag coefficients would be influenced by the free surface and hence the Froude Number should be considered. This would of course be true if the total drag coefficient were under consideration because the drag produced by the wave at the surface could be a very important part of the total drag. The investigation reported here deals with the local drag coefficient as determined from pressure measurements on the part of the cylinder that was completely submerged. It is believed that

the free surface had a negligible effect on the pressures measured at the cylindrical surface except for points near the free surface. It is, therefore, believed that the free surface had no effect on the results reported here.

Apparatus.—Control of the velocity distribution was obtained with a variable shutter gate. Variation of the gate opening from top to bottom permitted the resistance to be varied from a maximum near the bottom to a minimum at the

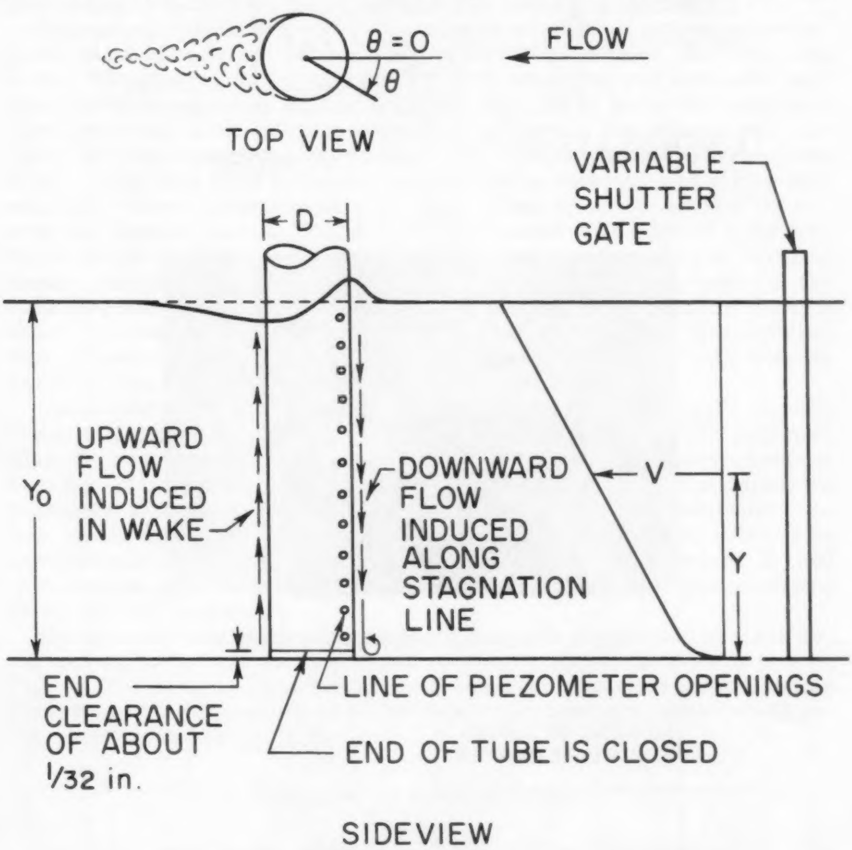
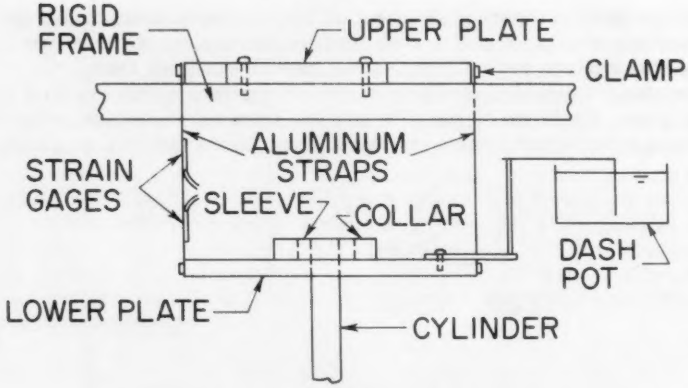


FIG. 1.—DIAGRAM OF TEST ARRANGEMENT

surface. The effectiveness of the gate in producing a linear velocity variation was enhanced by the inclusion of a serrated screen.

The cylinder was mounted to an aluminum plate suspended by four thin aluminum straps from a rigid frame. (Fig. 2) The lower plate was free to deflect as force was applied to the cylinder. Deflection of the lower plate introduced flexural stresses in the aluminum straps, the strain from which was picked up by a system of electrical strain gages, thereby enabling total force measurements to be obtained.



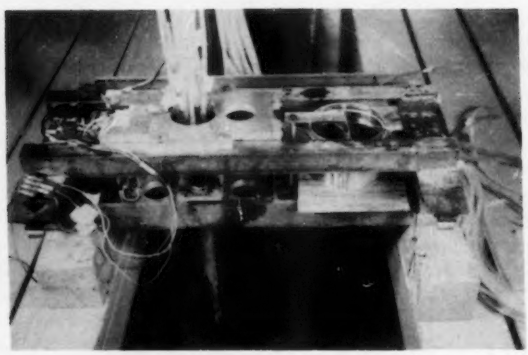


FIG. 2.—CYLINDER MOUNTING ASSEMBLY

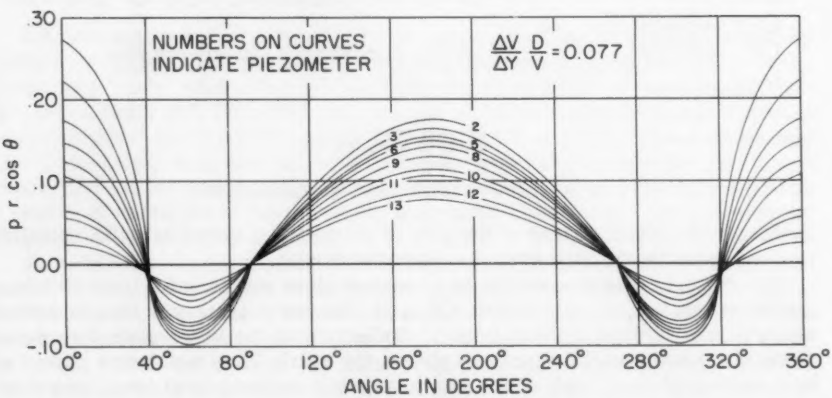


FIG. 3

The cylinder was attached to the lower plate by a sleeve fitted into a collar fixed to the lower plate. It was possible to rotate the cylinder through 360° about its longitudinal axis. Located along a vertical line on the cylinder were 24 piezometers, each of which was connected internally to a 1/8 in. copper tube. The tubes were brought out through the top of the cylinder and connected to a manometer board from which the local pressure was measured for each piezometer. Fig. 2 shows the cylinder and the amounting assembly.

Experimental procedure.—The experimental procedure involved measurements of the velocity distribution, local pressure distribution, and total drag force. The longitudinal velocity profiles were measured with a Prandtl pitot tube. Several transverse profiles were also obtained to insure that side wall effects were negligible. Pressure distributions around the cylinder were obtained with the piezometers by rotating the cylinder through 360° in increments of 30°. Total drag force measurements were taken several times during each run. All of these measurements were taken at four different values of the velocity gradient parameter. Total drag measurements were made as a check on the integrated pressure measurements. The measured total drag included, of course, the wave drag at the free surface. An approximate evaluation of the wave drag was made, based on the profile of the wave surface around the cylinder. The sum of the estimated wave drag and the pressure drag obtained from integration of the pressure measurements agreed satisfactorily with the total drag force as measured by the electrical strain gages.

The evaluation of the local drag coefficient at various points along the cylinder was determined from the pressure distributions and the measured velocity profiles. To illustrate the method of determining the local drag coefficient, consider a section of the cylinder. The pressure distribution as determined from the piezometer at the section was integrated to find the force at the particular section. As the functional relation between the pressure and the angle could not be expressed mathematically, Eq. 4 was evaluated by graphical integration. Typical of the curves used for this integration are those of Fig. 3. This plot includes a curve for each piezometer.

Having completed the integration of the pressure distribution on a section, the corresponding local velocity was taken from the plot of the velocity profile, Fig. 4. The local drag coefficients were computed at various positions along the cylinder for different values of the velocity gradient parameter in the manner described above.

RESULTS OF INVESTIGATION

Local drag coefficient.—The velocity gradient very definitely affects the local drag coefficient. The variation that exists may best be seen in Fig. 5. Here the local drag coefficient is plotted against the distance from bottom in feet for different values of the gradient parameter. If the velocity distributions were uniform, there would be no variation in the local drag coefficient as the velocity and pressure distribution would be constant along the cylinder. This curve indicates that the local drag coefficient does vary with position and also with the value of the slope of the velocity profile.

Consider first the variation with position along the cylinder. Fig. 5 indicates that the local drag coefficient is a maximum at the bottom of the cylinder and a minimum near the top. With reference to the velocity gradient, this corresponds

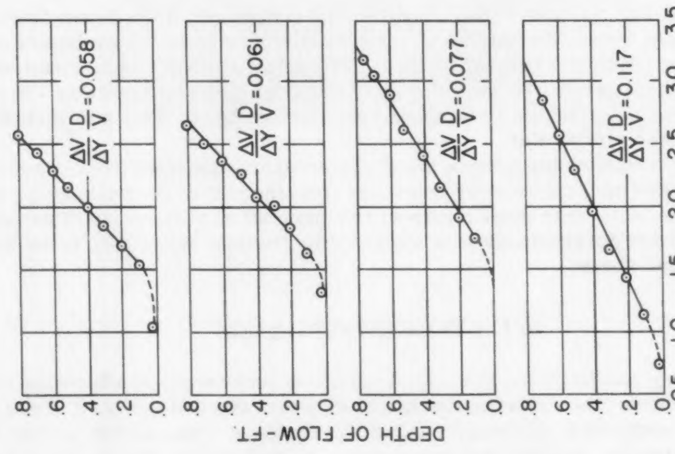


FIG. 4.—VELOCITY PROFILES

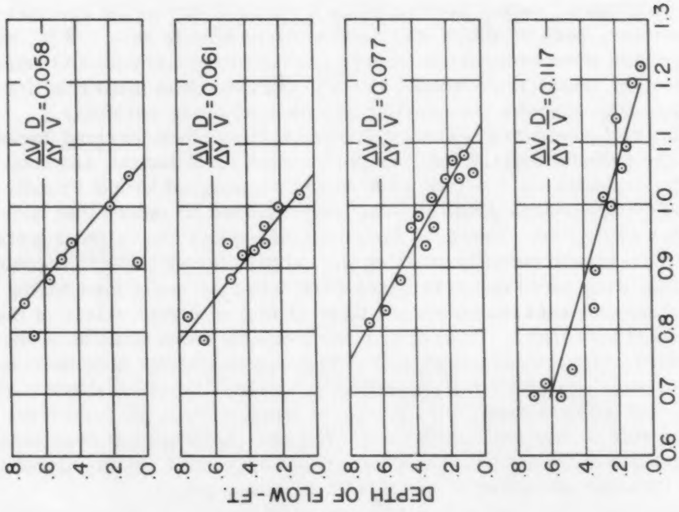


FIG. 5

to a high coefficient at the low-velocity end and a low coefficient at the high-velocity end. This variation in the drag coefficient was, as anticipated and previously described, in terms of the secondary flows.

There is, also, a variation of the local drag coefficient with the steepness of the velocity gradient. From the curves in Fig. 5, it can be seen that as the slope parameter increases from 0.058 to 0.117, the variation of the local drag coefficient becomes more pronounced. This follows from the fact that the greater the velocity gradient, the greater is the pressure variation in the wake zone, and the greater is the upward flow in the wake zone.

To illustrate the upward flow that exists in the wake, a dye was injected through one of the copper tubes and allowed to issue from a piezometer located in the wake zone. The result of this qualitative demonstration is presented in Fig. 6. The dye jet may be seen shooting out from the piezometer for a distance of about 1/4 of the cylinder diameter and then turning upward where it hugs the back of the cylinder. Although there is some subsequent diffusion of

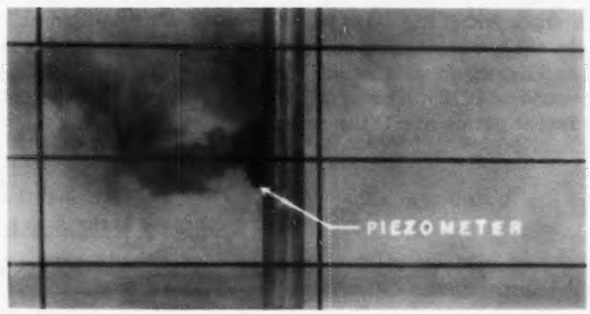


FIG. 6.—UPWARD FLOW IN WAKE.

the dye into the surrounding fluid as it is swept downstream, the existence of the upward flow is emphasized by the fact that only a trace of the dye appears below the piezometer opening for the downstream region.

Average drag coefficient.—In general, it may be concluded that the average drag coefficient for a cylinder with a longitudinal velocity gradient is less than the conventional value of 1.2 for the chosen range of Reynolds Numbers. The primary reason for this is that the flow is not two-dimensional. The presence of the longitudinal flow in the wake zone very definitely indicates three-dimensional flow characteristics.

The values of the average drag coefficient agree quite well with the approximate values given for cylinders of finite length. For values of $5 < \frac{L}{D} < 20$, the average drag coefficient varies from 0.74 to 0.90. (3) The results of this investigation more nearly approach these values at the steep velocity gradients, however, the causes are not directly related.

SIGNIFICANCE FOR ENGINEERING PROBLEMS

The investigation has shown that the longitudinal velocity gradient can effect the value of the drag coefficient and that a longitudinal flow is induced in the

wake of the cylinder which appears to be related to the effect. It is of interest to compare the values of the velocity gradient parameter used in the investigation with values that might be expected in some engineering problems. In the experimental investigation the parameter $\Delta V/\Delta Y\ D/V$ had the following values: 0.117, 0.077, 0.061, and 0.058.

For comparison, some values of the velocity gradient parameter were calculated for waves based on laboratory measurements of particle velocities in waves by T. A. Marlowe. (4) If it is assumed that a pier diameter is 0.05 times the water depth, the wave length is 2.5 times the water depth, the double amplitude is 0.11 times the water depth, and the velocity gradient at mid-depth is used, the gradient parameter will be 0.10. Use of the velocity gradient at 0.8 depth and all other factors the same gives a gradient parameter of 0.135. In a wave of greater height, that is with double amplitude 0.145 times the water depth, and using the velocity gradient at 0.8 depth the gradient parameter is 0.25.

If, for simplicity, one assumes that the wind velocity distribution near the ground may be expressed as the 1/6 power of the elevation, (1) an approximate value of the gradient parameter may be computed for a stack. If one considers a 15-ft diam stack 100 ft high, and bases the gradient parameter on the gradient at mid-height the gradient parameter will be 0.05. If one considers the lower portion where the gradient is steeper, for example, the lower 40 ft, the gradient parameter would be 0.125. These values indicate that the gradient parameters used in the investigation are in the range to be significant for some engineering problems.

CONCLUSIONS

In this exploratory investigation of the velocity gradient effect, test conditions were simplified in the following ways:

1. The ratio of cylinder diameter to water depth was essentially constant.
2. The Reynolds number range was chosen so that the conventional drag coefficient was essentially constant.
3. The velocity variation was approximately linear giving a constant value for the gradient at all points along the cylinder.

Based on the results obtained under these restrictions, the following conclusions appear to be justified:

1. A longitudinal velocity gradient in the flow approaching a circular cylinder induces longitudinal currents in the wake zone and along the upstream element, thus, altering the local drag coefficient at various positions along the cylinder.
2. The local drag coefficient is reduced at sections subject to highest stream velocity due to the rise in pressure as fluid is supplied into the wake from the regions of lower stream velocity.
3. The magnitude of the above effect increases as the magnitude of the velocity gradient increases.

These exploratory studies have indicated that the velocity gradient effect is significant in a number of problems of engineering importance, and has opened up several questions requiring further investigation. Further study is anticipated to evaluate the influence of some of the factors that were excluded in order to simplify this exploratory investigation.

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UNSTEADY FLOW OF GROUND WATER INTO A SURFACE RESERVOIR[a]

By William Haushild[1] and Gordon Kruse[2]

SYNOPSIS

Prediction of the water table position and the amount of water discharged where ground water is flowing from an aquifer to a surface reservoir has not been exact. The nonlinear partial differential equation that describes the shape of the water table is difficult to solve.

Approximate solutions obtained by two different methods for the nonlinear equation are presented. Both approximate solutions agree better with experimental results than does the exact solution of the simplified linear equation for the flow.

INTRODUCTION

The solution of one-dimensional boundary problems in ground water flow has many applications. For example, the increment of water added to bank storage when a surface reservoir is filled, or taken from bank storage when a reservoir is emptied, can be considerable and the amount is important in a determination of the reservoir hydrology. The same solution also applies to

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a Joint contribution from Quality Water Branch, U. S. Geol. Survey, U. S. Dept. of Interior and Soil and Water Conservation Research Div., Agric. Research Service, U. S. Dept. of Agric.

1 Hyd. Engr., Quality Water Branch, U. S. Geol. Survey, Stationed at Colorado State Univ., Fort Collins, Colo.

2 Agric. Engr., Soil and Water Conservation Research Div., Agric. Research Service, U. S. Dept. of Agric., Stationed at Colorado State Univ., Fort Collins, Colo.

certain unsteady cases of ground water flow caused by the digging of drains and to bank storage adjacent to streams that undergo changes in water level. This paper presents four approximate solutions for the problem. Solutions I and III are first approximations in which the effect of the unwatering of the aquifer on the area available for the flow of ground water is neglected. Solutions II and IV are second approximations that take the effect of unwatering into account.

The solutions presented in this paper are based upon the following assumptions:

1. The saturated aquifer is of infinite extent and overlays an impermeable layer of zero slope.
2. The Dupuit-Forchheimer assumptions hold.
3. The change in reservoir water level is instantaneous.
4. The aquifer is composed of an isotropic and homogeneous material.

SOLUTION I

Consider the flow into a reservoir as shown in Fig. 1. The flow through a cross section of unit width at the distance x from the origin is

$$Q = K\left(d + \frac{H}{2} + h\right)\frac{\partial h}{\partial x} \quad \text{(1)}$$

in which K is the permeability of the aquifer.
If we let

$$D = d + \frac{H}{2} \quad \text{(2)}$$

then

$$Q = K\left(D + h\right)\frac{\partial h}{\partial x} \quad \text{(3)}$$

The equation of continuity for the strip of width dx and time increment dt is

$$\frac{\partial Q}{\partial x}\, dx\, dt = V\, \frac{\partial h}{\partial t}\, dt\, dx \quad \text{(4)}$$

in which V is the specific yield of the aquifer.
Or, by use of Eq. 3

$$K\left(D + h\right)\frac{\partial^2 h}{\partial x^2} + K\left(\frac{\partial h}{\partial x}\right)^2 = V\,\frac{\partial h}{\partial t} \quad \text{(5)}$$

By letting $\alpha = \frac{K D}{V}$, Eq. 5 becomes

$$\alpha\,\frac{\partial^2 h}{\partial x^2} + \frac{\alpha}{D}\left(\frac{\partial h}{\partial x}\right)^2 = \frac{\partial h}{\partial t} - \frac{\alpha}{D}\, h\,\frac{\partial^2 h}{\partial x^2} \quad \text{(6)}$$

This differential equation is nonlinear in form. If d is much larger than H, h can be discarded from Eq. 3. The continuity equation (Eq. 4) then gives the following linear differential equation:

$$\alpha\,\frac{\partial^2 h}{\partial x^2} = \frac{\partial h}{\partial t} \quad \text{(7)}$$

Eq. 7 is solved by Carslaw and Jaeger[3] as it applies to the flow of heat in solids. Ferris[4] applied Eq. 7 to the case of constantly discharging surface drains. Todd[5] solved an equation similar to Eq. 6, by numerical methods.

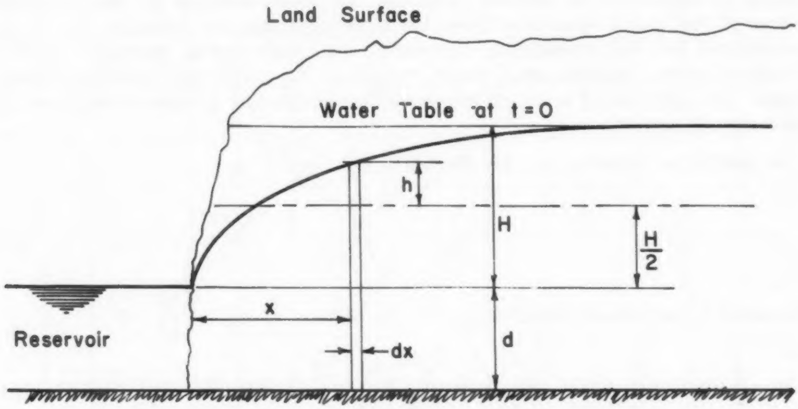

FIG. 1.—GROUND WATER DISCHARGE INTO A BODY OF SURFACE WATER

The results obtained concurred with data obtained from a Hele-Shaw viscous flow model.

The solution of Eq. 7 given by Carslaw and Jaeger is

$$h_1 = -\frac{H}{2} + H\,\phi \quad \text{(8)}$$

in which ϕ, the error function, is

$$\phi = \frac{2}{\sqrt{\pi}} \int_0^{x/\sqrt{4\,\alpha\,t}} e^{-u^2}\,du \quad \text{(9)}$$

The integral is tabulated elsewhere.[6]

This is solution I. When the drawdown is small compared to the thickness of the aquifer, solution I is an approximate solution of Eq. 6. It satisfies the boundary and initial conditions:

for $x = 0 \quad t > 0 \quad h_1 = -\frac{H}{2}$

$x \to \infty \quad h_1 = \frac{H}{2}$

$t = 0 \quad x > 0 \quad h_1 = \frac{H}{2}$

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6 "Tables of the Error Function and its Derivative," Natl. Bur. of Standards Applied Math. Series No. 41, October, 1954.

SOLUTION II

A second approximation to the true solution of Eq. 6 may be obtained by applying the method of Picard.[7] This method uses each successive approximation to approach the correct solution. To obtain solution II, the nonlinear terms of Eq. 6 are computed from the first approximate solution, Eq. 8, and substituted into Eq. 6 as known functions. The differential equation, as thus modified, is then solved, again being subject to the initial and boundary conditions. The process of solution requires that particular integrals be found for the known functions.

A particular integral p_1, for the term $-\frac{\alpha}{D}\left(\frac{\partial h_1}{\partial x}\right)^2$ is

$$p_1 = -\frac{h_1^2}{2\,D} \qquad (10)$$

Similarly a particular integral p_2, for the term $-\frac{\alpha}{D}\,h_1\,\frac{\partial^2 h_1}{\partial x^2}$ is

$$p_2 = -\frac{1}{2\,D}\,\frac{\partial h_1}{\partial x}\int h_1\,dx \qquad (11)$$

The expanded form of this particular integral is

$$p_2 = -\frac{H}{2}\left[-\frac{H}{2\,D}\,\frac{2}{\sqrt{\pi}}\,\frac{x}{\sqrt{4\,\alpha\,t}}\,e^{-\frac{x^2}{4\,\alpha\,t}} + \frac{H}{D}\,\frac{2}{\pi}\,e^{-\frac{2\,x^2}{4\,\alpha\,t}} + \frac{H}{D}\,\frac{2}{\sqrt{\pi}}\,\frac{x}{\sqrt{4\,\alpha\,t}}\,e^{-\frac{x^2}{4\,\alpha\,t}}\,\phi\right] \qquad (12)$$

An approximate solution of the modified form of differential Eq. 6 that satisfies the initial and boundary conditions is

$$h_2 = p_1 + p_2 + q \qquad (13)$$

This is solution II, in which q is the first approximation, Eq. 8, altered to correct h_2 for the initial and boundary conditions.

$$q = -\frac{H}{2}\left[1 - \left(\frac{H}{4\,D} + \frac{2}{\pi}\,\frac{H}{D}\right)\right]\left[1 - \phi\right] + \frac{H}{2}\left[1 + \frac{H}{4\,D}\right]\phi$$

A plot of Eq. 13 is shown in Fig. 2 with the dimensionless parameter h/H as ordinate and $x/\sqrt{4\,\alpha\,t}$ as abscissa. A complete drawdown is assumed so that the solution can be compared with experimental data. The solution for the linear case, Eq. 8, is also shown in Fig. 2.

Keller and Robinson[8] obtained measurements of the drawdown curve in a laboratory experiment in a sand filled flume. Water was drained from the sand

7 "Memoire sur la theorie des equations aux derivees partielles et la method des approximations successives," by Emile Picard, Liouville's Jour. de Methematiques, Series 4, Vol. 6, 1890, pp. 145-210.

8 "Model Study of Interceptor Drains, by Jack Keller and A. R. Robinson, Proceedings, ASCE, September, 1959, pp. 25-40.

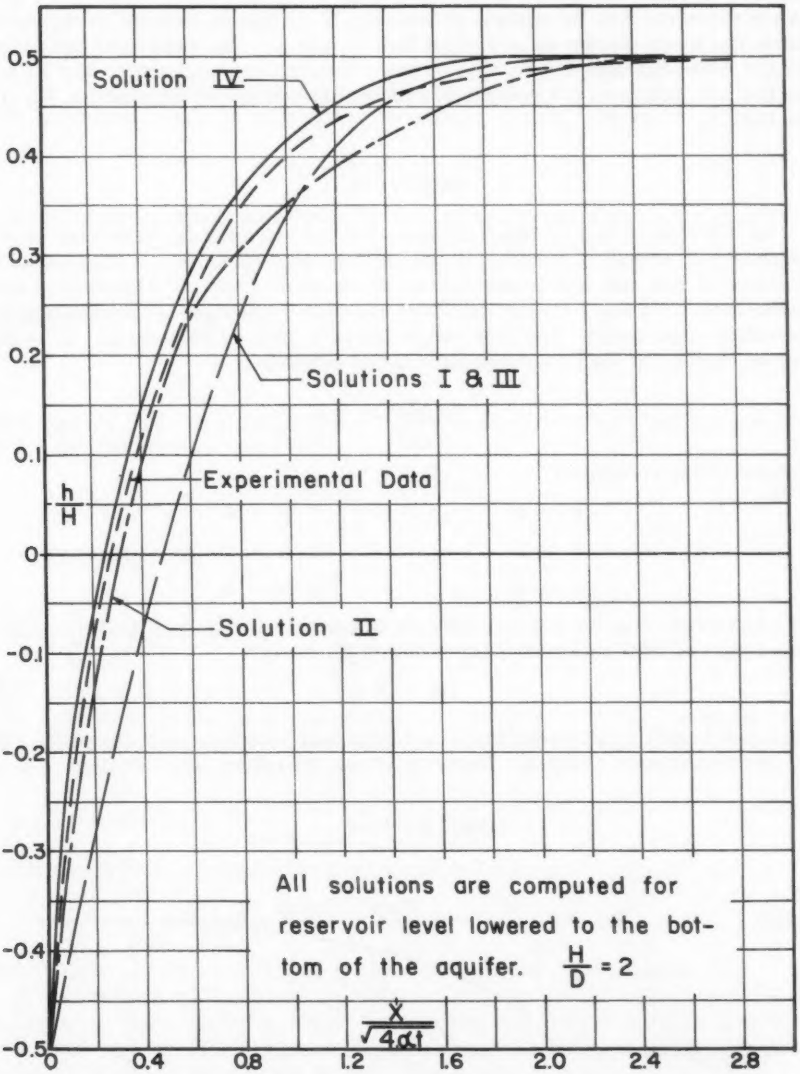

FIG. 2.—APPROXIMATE SOLUTIONS OF GROUND WATER EQUATIONS AND EXPERIMENTAL DATA

through a 4-in. perforated pipe located at the floor of the flume. The entire flow of water was intercepted by the drain. These conditions simulated flow into a surface reservoir where the water is drawn down to the bottom of the aquifer ($d = 0$). The shape of the water table was determined before the draw-down curve reached the upstream boundary of the flume. Results were, therefore, the same as for an aquifer of infinite extent. The sand used had values of $K = 0.034$ fps, and $V = 25.5\%$ and was practically homogeneous. The data of Keller and Robinson are compared with the theoretical approximation, Eq. 13, in Fig. 2.

SOLUTION III

An alternative second approximation for the bank storage case may be obtained by a method of equating flows. A first approximation is obtained as in solution II and the unit discharge, Q, is computed from it. This flow is then substituted into Eq. 3 and an improved solution is obtained by integrating the resulting expression. The first approximation will be solution III. It is obtained by solving the linearized differential equation

$$\alpha \frac{\partial^2 h}{\partial x^2} = \frac{\partial h}{\partial t} \quad \text{. . . . .} \quad (14)$$

subject to the conditions

$$\begin{array}{lll} x = 0 & t > 0 & h = 0 \\ t = 0 & x > 0 & h = H \\ x \to \infty & & h \to H \end{array}$$

The significance of the notation used in this case is shown in Fig. 3.
The required first approximation, solution III, is then

$$h_3 = H\,\phi \quad \text{. . . . .} \quad (15)$$

Solutions I and III satisfy the same equations and boundary conditions with only the reference point changed. These solutions, therefore, are identical.

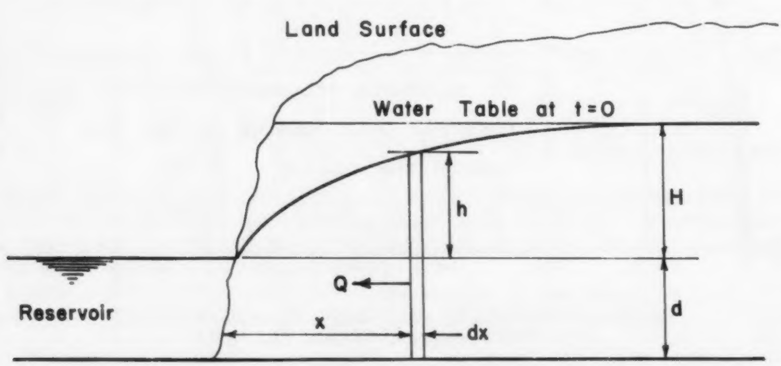

FIG. 3.—GROUND WATER DISCHARGE INTO A BODY OF SURFACE WATER

SOLUTION IV

A first approximation to the flow Q_1 through a cross section of unit width at x is

$$Q_1 = K D \frac{\partial h_3}{\partial x} \quad \ldots\ldots\ldots\ldots\ldots \quad (16)$$

in which $D = d + \frac{H}{2}$ and h_3 is given by Eq. 15.
If the drawdown is taken into account, the expression for flow is

$$Q = K (d + h) \frac{\partial h}{\partial x} \quad \ldots\ldots\ldots\ldots\ldots \quad (17)$$

To obtain a closer approximation of h, Eqs. 16 and 17 are arbitrarily equated:

$$K (d + h) \frac{\partial h}{\partial x} = K D \frac{\partial h_3}{\partial x} \quad \ldots\ldots\ldots\ldots \quad (18)$$

An integration with respect to x yields

$$\frac{(d + h)^2}{2} = D h_3 + C_1 \quad \ldots\ldots\ldots\ldots \quad (19)$$

The constant C_1 may be evaluated for the boundary condition that $h = 0$ when $x = 0$. The value so obtained is

$$C_1 = \frac{d^2}{2} \quad \ldots\ldots\ldots\ldots\ldots \quad (20)$$

Then, by substitution

$$(d + h)^2 = d^2 + 2 D h_3 \quad \ldots\ldots\ldots\ldots \quad (21)$$

Solution IV is obtained from this relation in the form

$$h_4 = \sqrt{2 D h_3 + d^2} - d \quad \ldots\ldots\ldots\ldots \quad (22)$$

This solution satisfies the initial condition $h = H$ when $t = 0$. If h_4 is measured from the same datum as h_2 so that they may be compared directly, Eq. 21 becomes

$$h_4' = \sqrt{2 D h_3 + d^2} - d - 0.5 H \quad \ldots\ldots\ldots\ldots \quad (23a)$$

For the case of a complete drawdown it takes the form

$$h_4' = \sqrt{H h_3} - 0.5 H = H \sqrt{\phi} - 0.5 H \quad \ldots\ldots\ldots \quad (23b)$$

This equation is shown on Fig. 2 for comparison with experimental data.

A comparison of these solutions shows that the second approximations II and IV show closer agreement with the experimental data than do the first approximations I and III. Fortunately, the best agreement is obtained from solution IV, which is the easiest to use. The comparison is made here with the extreme case of complete drawdown.

All the solutions are approximations based on the assumption that d is very large compared to H. This assumption is least valid for the case of complete drawdown. Therefore, the results from using this solution with partial drawdowns should be good.

Solutions I through IV were derived for the case of inflow into a reservoir upon lowering of the reservoir water level. However, the same equations and solutions apply to the case of outflow if Q is considered to be negative for flow into the aquifer.

COMPUTATION OF α

Curves similar to those of Fig. 2 can be used to compute α for a given aquifer if values of h, H, d, x, and t have been measured in the field. From these values H/D can be computed and curves drawn for solution I or solution IV using h/H as ordinate and $x/\sqrt{4\,\alpha\,t}$ as abcissa. For a given h/H, α can be computed from the measured value of $x/\sqrt{t}$ and the value of $x/\sqrt{4\,\alpha\,t}$ obtained from the curve.

TABLE 1.—ERROR IN DETERMINATION OF α

h/H	Solution I, in percent	Solution IV, in percent
-0.41	88	84
-0.12	77	36
+0.06	60	24
+0.42	21	17

Solutions I and IV were used to determine α by the above method for several of the points from Keller and Robinson's data.[8] These values were compared with the value of α computed from laboratory measurements of K, D, and V. Table 1 indicates the percentage error in α as determined by these two solutions.

Journal of the
HYDRAULICS DIVISION
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DISCUSSION

DISCHARGE FORMULA FOR STRAIGHT ALLUVIAL CHANNELS[a]

Discussion by R. J. Garde and John L. Bogardi

R. J. GARDE,[1] A.M. ASCE.—The authors are to be complimented for their work concerning the discharge formula for the straight alluvial channels. They have shown that if the mean velocity of flow in a straight alluvial channel is given by the equation

$$V = C_a \; R^x \; S^y \qquad (1)$$

then C_a, x, and y are not constants but they depend on the size of the bed material and the regime of flow. According to the analysis carried by Messrs. Liu and Hwang, C_a can vary between 11 and 287, x can vary between 0.35 and 0.71, and y can vary between 0.30 and 0.57. For the large volume of flume data, the correlation obtained seems to be remarkable.

During 1956-59, the writer carried out certain investigations[2] concerning the velocity distribution and the sediment transport in alluvial channels. One of the significant findings of these investigations was the fact that velocity distribution and sediment transport are very much dependent on whether the bed of the channel is plane, or has ripples and dunes on it. The work of Messrs. Liu and Hwang confirms this finding.

In order to find out whether this new formula proposed by Messrs. Liu and Hwang can be applied to the natural channels and rivers, the writer has carried out certain preliminary analyses and the results are reported herein. The data used for this analysis cover wide ranges of the discharge Q, the mean velocity V, the hydraulic radius R, and the bed material size d. Table A gives a summary of these data.

The procedure followed was:

1. To determine the regime of flow according to $\dfrac{V_*}{W} - \dfrac{V_* d}{\nu}$ Criterion.

Note.—This paper is a part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. HY 7, July, 1960.

[a] November, 1959, by H. K. Liu and S. Y. Hwang.

[1] Lectr. in Civ. Engrg., Univ. of Roorkee, Roorkee (U.P.), India.

[2] "Total Sediment Transport in Alluvial Channels," by R. J. Garde, Ph. D. Dissertation submitted to Colorado State Univ., Fort Collins, Colo., January, 1959.

2. To determine x, y and C_a.
3. To compute the mean velocity of flow by the use of Eq. 1.

It was found that several points from these field data fell in the region of 'Transition' of Fig. 1 (given by the authors). When this predicted regime was used and the corresponding values of C_a, X and Y were chosen, the values of the mean velocity could be computed. It was found:

a. For points falling in the dune regime, the observed velocity, most of the time, was greater than the computed velocity.

b. For the points falling in transition regime the observed velocity was less than the calculated velocity.

Considering, for example, the Punjab Canal's data, 8 points out of 42 fell in the transition regime and for these 8 points the computed velocities were

TABLE A.—SUMMARY OF DATA

River or Canal and Country. (1)	Observation Points (2)	Range of d, in mm. (3)	Range of R, in ft. (4)	Range of Q, in cfs. (5)	Range of Q, in cfs. (6)
Punjab Canals (India)[a]	42	0.15 - 0.43	0.9- 9.5	1.1- 3.3	-
Sind Canals (India)[a]	20	0.02- 0.17	4-12	1.5- 3.4	300- 9000
Ganga Canal (India)	8	0.43	11.5- 12.4	3.4- 4.0	7400- 9700
Indus River (India)[b]	6	0.97	17.8- 26.8	6.8- 7.0	3,35000- 5,80700
Tiber River (Italy)[b]	11	0.3 - 9.5	4-13.4	2.0- 3.1	4500- 11600
Rakaia (New Zealand)[b]	1	101.6	3.5	5.0	4500
Opihi (New Zealand)[b]	1	50.8	0.7	1.1	81
Wanganui (New Zealand)[b]	1	19.0	3.90	1.4	1630
Mountain Creek (U.S.A.)[c]	8	0.87	0.3- 0.6	1.5- 2.3	-
Canals (U.S.A.)[a]	2	7.0 - 7.6	4.6- 6.0	2.3- 2.6	750- 880

[a] "Theory and Design of Stable Channels in Alluvial Materials," by D. B. Simons, Ph. D. Dissertation submitted to Colorado State Univ., Fort Collins, Colo., May, 1957.
[b] "Slope Discharge Formulae for Alluvial Streams and Rivers," by E. C. Schnackenberg, Proceedings, New Zealand Inst. of Civ. Engrs., Vol. 37, 1951.
[c] "Bed Load Transportation in Mountain-Creek," by H. A. Einstein, U. S. Dept. of Agric. SCS-TP 55, August, 1944.

greater than the observed velocities by 35% to 125%. For the 34 points that fell in the dune regime, the computed velocities were always smaller than the observed velocities by 14% to 43%. Similar tendency was noticed in the case of Sind Canal's data.

One of the possible reasons for such high variation (in the case of data falling in 'transition' regime) could be that the regime predicted by $\frac{V_*}{W} - \frac{W\,d}{\nu}$ criterion is not correct. In fact, the authors have stated that this criterion is based on the flume data for which the Froude numbers are much larger than those for natural streams. The accuracy of the results obtained by the Liu-Hwang formula depends on the correct prediction of the regime and therefore the criterion for determining regime of flow becomes a very important factor.

As pointed out by the authors, $\frac{V_*}{W} - \frac{W\,d}{\nu}$ or $\frac{V_*}{W} - \frac{V_*\,d}{\nu}$ criteria predict the regimes of flow with fair degree of accuracy for the flume data. However, when applied to the natural river data with larger depths and flatter slopes, the criteria encounter serious difficulties. In the case of natural river data, because of high values of shear, $\frac{V_*}{W}$ attains a large value and thus the points fall in transition or antidune region. Actually these points belong to ripple dune region. This discrepency occurs because the Froude number is not taken into consideration while determining the regime.

From the point of view of dimensional considerations it can be shown that, as a first approximation, the ratio of height of dunes to length of dunes depends on $\frac{\tau_b}{\Delta \gamma_s\, d}$ and $\frac{V}{\sqrt{g\,D}}$. W. B. Langbein[3] and Z. Matunashi[4] have also shown that the Froude number is an important factor in the determination of regimes of flow in alluvial channels. Similarly, $\frac{\tau_b}{\Delta \gamma_s\, d}$ is important in the determination of velocity distribution in alluvial channels.[2] For these reasons one would expect that $\frac{\tau_b}{\Delta \gamma_s\, d}$ and F_r will be significant parameters for establishing the criterion for the determination of regimes of flow. This was done by R. J. Garde and M. L. Albertson[2, 5] and this criterion seems to be quite satisfactory. It is applicable to flume data as well as natural river data.

Since the Froude number, which is one of the parameters determining regimes of flow, contains the mean velocity of flow, this $\frac{\tau_b}{\Delta \gamma_s\, d} - F_r$ criterion cannot be used while determining the mean velocity of flow. However, this criterion can be used with advantage in the present case where the purpose is to study the applicability of equation 1 to natural rivers and canals.

For these reasons, the regimes of flow of all the data were determined according to $\frac{\tau_b}{\Delta \gamma_s\, d} - F_r$ criterion; several points which fell in the 'transition' regime according to $\frac{V_*}{W} - \frac{W\,d}{\nu}$ criterion, now fell in the ripple-dune regime. This made a considerable difference in the computed mean velocity of flow according to Eq. 1. Table B shows the comparison between the observed mean velocity and the computed mean velocity for the regime predicted by the two criteria.

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3 "Hydraulic Criteria for Sand Waves," by W. B. Langbein, Transactions, Amer. Geophysical Union, 1942, pp. 615-618.

4 Z. Matunashi, Personal letter.

5 "Characteristics of Bed Forms and Regimes of Flow in Alluvial Channels," by R. J. Garde and M. L. Albertson, Civ. Engrg. Sect., Report CER 59 RJG 9, Colorado State Univ., Fort Collins, Colo.

TABLE B.—COMPARISON BETWEEN OBSERVED AND COMPUTED MEAN VELOCITY

Canal or river	Observed V, in fps.	Calculated V, in fps	
		Transition regime	Ripple-dune regime
(1)	(2)	(3)	(4)
Sind Canals	2.41	4.70	2.52
	2.95	5.76	2.83
Tiber River	3.10	6.50	1.98
	2.73	6.45	1.92
	2.93	6.08	1.85
	2.74	5.90	1.81
	2.51	5.57	1.75

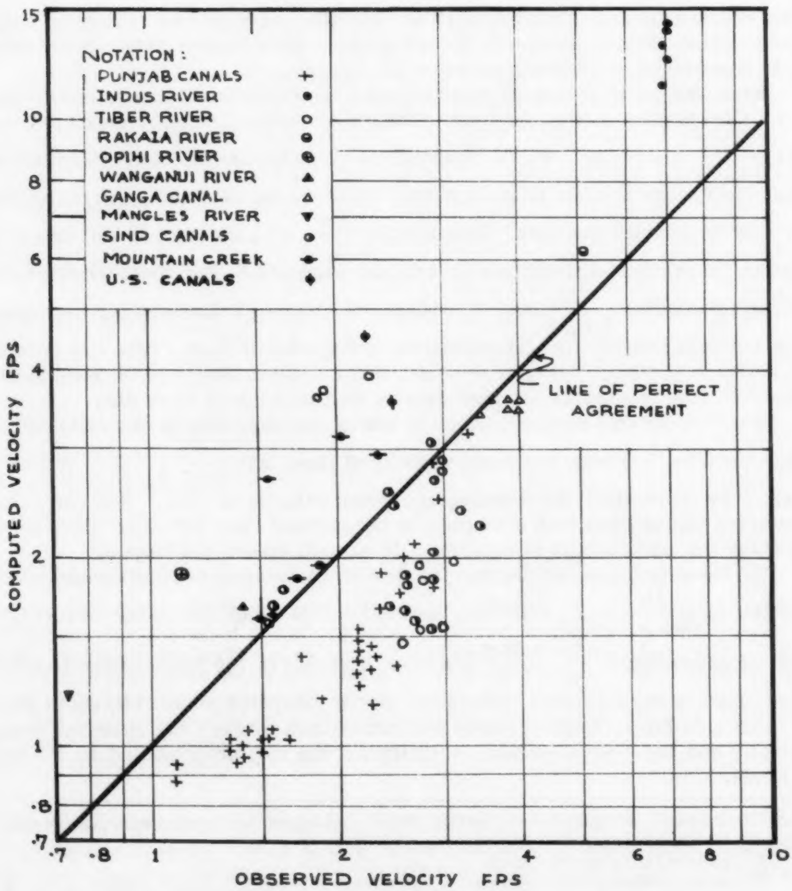

FIG. 1.—COMPARISON OF OBSERVED AND COMPUTED MEAN VELOCITIES.

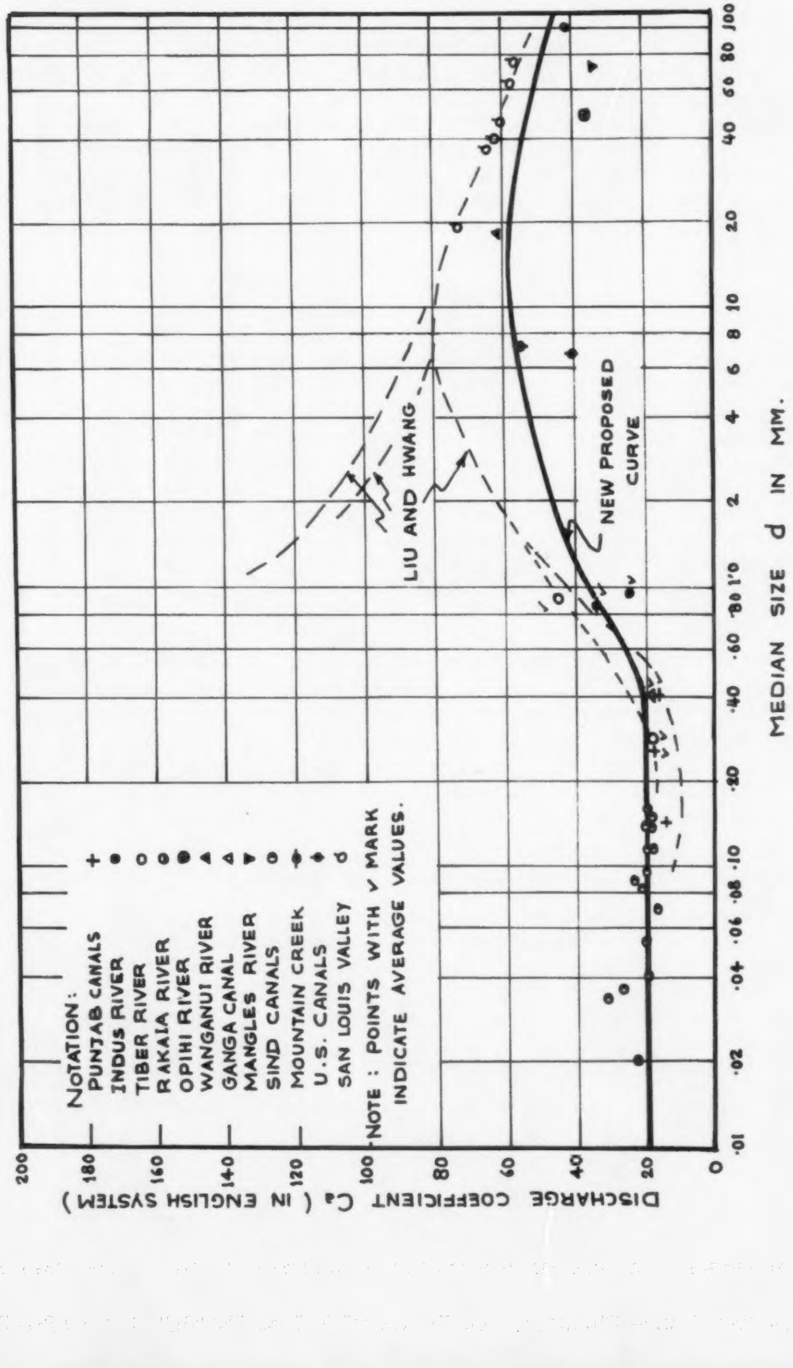

FIG. 2.—VARIATION OF C_a (IN ENGLISH SYSTEM) WITH d mm.

Thus it can be seen that computed velocities according to ripple-dune are in better agreement with observed velocities. Fig. 1 shows the comparison between observed mean velocity of flow and computed mean velocity for the various data. It can be seen that the general tendency is for the observed velocity to be greater than the computed mean velocity. From Fig. 1, it can also be seen that 35% of the data have less than 25% error.

By slight adjustment of the C_a - d curve a better agreement could be obtained. The following procedure was adopted. It was assumed that the values of x and y obtained from the authors' graphs were correct and that adjustment could be made for C_a values in such a manner that observed mean velocity of flow is equal to the calculated mean velocity of flow. Thus

$$C_a \text{ computed} = \frac{(V) \text{ observed}}{R^x \, S^y} \qquad (2)$$

New values of C_a were computed in this manner for all the data available to the writer. When there was more than one observation for the same size of bed material and the same regime of flow, the average new value of C_a was found. Fig. 2 shows the variation of C_a with d for the ripple-dune regime. Lane and Carson's data used by Messrs. Liu and Hwang are also plotted in Fig. 2. Tentatively a mean curve passing through these points is drawn. Up to the size of bed material 0.5 mm or 0.6 mm, this new curve is very close to the one proposed by the authors. However, for coarser material the proposed curve gives lower values of C_a than the one given by the authors' curve. More data are needed to be plotted on this figure, especially for coarser material, in order to confirm the location of the proposed curve.

JOHN L. BOGARDI.[6]—The attempt of the authors to investigate this extremely involved problem is very much appreciated. The writer agrees with the statement that with the effect of sediment transport upon resistance being unknown, empirical relationships rather than theoretical solutions should provide the correct approach. The soundness of this attitude is corroborated by the successful study of the authors.

An almost complete relationship between the three invariants $\frac{V_* d}{\upsilon}$, $\frac{w\,d}{\upsilon}$, and K is revealed by Figs. 4 and 7.

In these two figures, the points representing identical $\frac{w\,d}{\upsilon}$ values obviously always result from the same experiment, in which the particle size, the specific gravity of the sediment, and the water temperature are uniform, or in which there are but slight changes in the water temperature only. Each of the parallel straight lines in Figs. 4 and 7, sloping at $\Omega = 0.555$ and $\Omega = 0.565$, respectively, represent the result of an experiment presumably carried out under different conditions of observation. Nevertheless, a positive relationship can be established in Figs. 5 and 8 between the coefficient "A" and the invariant $\frac{w\,d}{\upsilon}$, which corroborates the general validity of the relationship, termed as empirical correlation by the authors.

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[6] Head, Hydr. Lab. of Research Inst. for Water Resources Development, Budapest, Hungary.

In the writer's opinion the explanation given by the authors as to the composition of the invariant group K is slightly ambiguous. According to this, the composition of K should be supported by the following considerations:

1. To conform with the existing knowledge of boundary resistance.
2. To correlate the data consistently.

Namely all the variables

$$\left(V_* = \sqrt{g\, R_b S}, \; \frac{\tau_b}{\left(\gamma_s - \gamma\right) d} = \frac{\gamma\, R_b\, S}{\left(\gamma_s - \gamma\right) d}, \right.$$

$$\left. S, \; \frac{R_b}{d}, \; F = \frac{V}{\sqrt{g\, R_b}} \right)$$

with the exception of the unknown velocity V are contained in the invariants $\frac{V_*}{\upsilon}$ and $\frac{w\, d}{\upsilon}$.

In principle, there could be no objection to using a variable, respectively invariant, differing essentially from K. Subsequently the authors themselves admit that the good applicability of the invariant group K (namely that the points representing identical $\frac{w\, d}{\upsilon}$ values should be straight lines) has been attained by the appropriate selection of values λ, m, and N.

In fact, a wide variety of solutions can be conceived for a function of the form

$$\frac{V}{V_*} = f\left(S, \frac{w\, d}{\upsilon}, \frac{R_b}{d}, \frac{V_*\, d}{\upsilon}, \frac{g\, d}{V_*^2}\right) \qquad (3)$$

The circumstance, that the above invariants will be involved in the function, has been illustrated by the authors themselves by the aid of dimensional analysis. On the basis of a few measured data the writer has thus temporarily determined the function of Eq. 1 with fair accuracy by six-variable correlation. It appears that with more data available the value of $\frac{V}{V_*}$, respectively of V could be determined graphically with very great accuracy.

Regardless of these considerations, the writer wishes to state that the authors have made very significant advances in this difficult problem. The writer agrees with the authors' statement concerning the exponents of x and y. In his opinion the method used for determining x and y is such as to already contain the effect of various regimes of movement. It is likely, therefore, that x and y are related to the invariants that are characteristic for movement.

The attitude of the authors towards the application of dimensionless analysis may give rise to misunderstanding. Actually, the use of dimensional analysis alone does not guarantee that there has not been any important variable omitted from the investigation.

In connection with the authors' comments on Fig. 1 in its original and present form, the writer would like to point out, that by representing the relationship

$$\frac{g\,d}{V_*^2} = \beta\, d^N \qquad (4)$$

the tractive force can in all probability be determined in an even simpler form.

THE SETTLING PROPERTIES OF SUSPENSIONS[a]

Discussion by Charles G. Gunnerson

CHARLES G. GUNNERSON,[1] F. ASCE.—The author has pointed out that the analyses of settling velocities of various sewage effluents shown in his Fig. 2 were made in connection with studies of marine disposal in the vicinity of Los Angeles. During the same period observations were made of the behavior of these effluents in the receiving waters. The results of the marine studies have been previously reported.[2] It is sufficient here to summarize the data relating to the sedimentation of effluent suspended solids in sea water.

The sanitary significance of sedimentation of suspended solids from the effluent of the Hyperion Treatment Plant lies in the concurrent reduction of enteric bacteria populations in surface waters. Accordingly, observations were made of the rate of disappearance of coliform bacteria from the surface. The probable effects of dilution and mortality were determined and the effect of sedimentation was computed.

It was found that, on the average, coliform bacteria were removed from surface waters by sedimentation at rates such that 90% removals occurred as follows:[3]

Hyperion secondary effluent (plus elutriation overflow)—21.0 hr.

Hyperion primary effluent (plus elutriation overflow)—5.3 hr.

Orange County Sanitation Districts primary effluent—2.0 to 2.4 hr. (A range of values is given because of some uncertainty in determining the effect of dilution).[3]

These results are based upon samples taken from the upper 1-ft layer of the water over periods of up to 1 day.

In order to compare the field results with the author's settling tube studies, the data in his Fig. 2 have been replotted to show suspended solids reductions as a function of time by removing the 40 cm depth in the z/t function. It is assumed that the sedimentation velocities represent removals from a slice at the 40 cm depth. It is further assumed that such removals can be compared, at least on a gross basis, with removals from the essentially homogeneous upper 1-ft layer of the ocean.

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[a] December, 1959, by Ronald T. McLaughlin Jr.

[1] Civ. Engr., Bur. of Sanitation, City of Los Angeles, Calif.

[2] "Sewage Disposal in Santa Monica Bay, California," by C. G. Gunnerson, Proceedings, ASCE, Vol. 84, No. SA 1, February, 1958, pp. 1-28, and Transactions, ASCE, Vol. 124, 1959, pp. 823-851.

[3] "Studies on Coliform Bacteria Discharged from the Hyperion Outfall, Final Bacteriological Report," by S. C. Rittenberg, Allan Hancock Foundation, Univ. of Southern California, Los Angeles, Calif., 1956.

Fig. 1 shows both the laboratory and field data for the removal of suspended solids with time for Hyperion and Orange County Sanitation Districts effluents. Although there are minor differences in the groups of effluents tested, the rates of sedimentation in the ocean are clearly greater than those in the settling tubes. Qualitatively, at least, the effect of flocculation of the colloidal fraction of the suspended solids appears to account for the greater part of solids removal in the ocean after the first few hours.

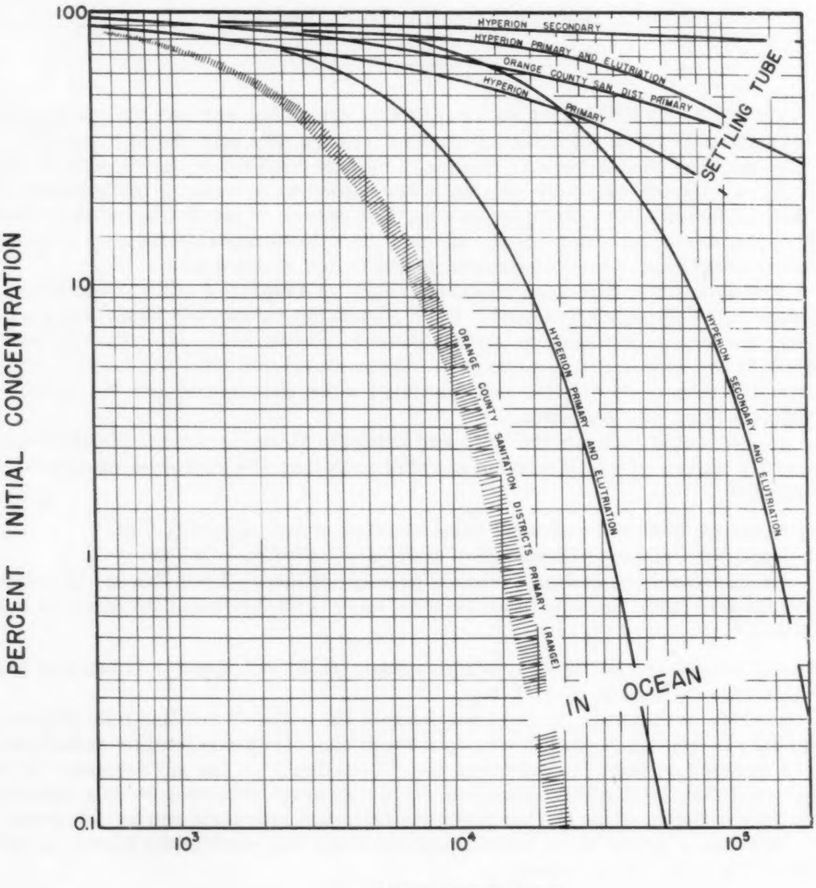

FIG. 1.—COMPARISON OF SEDIMENTATION RATES IN SETTLING TUBES AND THE OCEAN.

The flocculation of stream-carried suspended solids discharged into salt water has been studied by marine geologists for many years. K. O. Emery[4] considers that the variable composition of source rocks and diagenetic changes after deposition on the ocean floor are probably more important factors in the distribution of clay minerals in marine sediments off the California coast than

[4] "The Sea Off Southern California," by K. O. Emery, Wiley, 1960.

are differential flocculation and deposition. Nevertheless, Tj van Andel and H. Postma[5] have recently used the results of V. G. Whitehouse, et al.,[6,7,8,9] in explaining the near- and off-shore distribution of illite and montmorillonite, respectively, in the Gulf of Paria.

Some of the aspects of Whitehouse's results in studies of differential flocculation and settling of clay minerals appear to be applicable to sedimentation of effluent solids in the ocean. He found that the median settling velocities increased by about two orders of magnitude upon flocculation in saline waters with 18 parts per thousand $^0/_{00}$ chlorinity for the various clays tested. Initial settling rates increase by about 2% and 22% for kaolinite and illite, respectively, as the chlorinity is increased from 0.5 to about 3 $^0/_{00}$ and are unaffected by further increases in chlorinity. In contrast, as the chlorinity increases from 0.5 to 18 $^0/_{00}$, initial settling rates for montmorillonite are increased by factors of from 3 to 37 in a temperature range of 26° to 6° C. However, the velocities are still of the order of 1/10 those of the other clays. The effect of a temperature reduction from 21° to 9° C (this is about the seasonal range in Santa Monica Bay in the upper 200 ft of water) is to decrease settling rates of the three clays by about 30%. Ions of heavy metals increase the settling rates; the highest increase of 85% was reported for montmorillonite in the presence of 4×10^{-4} molal Fe^{3+}. The presence of carbohydrates increases the settling velocity of montmorillonite by 35% and 25% at 0.5 and 18 $^0/_{00}$ chlorinity, respectively. Humic acids or proteins cause decreases in velocities of up to 1% for montmorillonite and up to 30% in kaolinite. As the clay concentration decreases, there is a small but significant increase in settling velocity. Other factors which influence settling are pH, clay concentration gradients, and the initial state of division of the clay.

The fate of dispersed organic solids is not as well characterized. Some of the organics are undoubtedly absorbed onto the inorganic particles, some may be flocculated and settled, while the remainder may remain dispersed for a considerable time. D. L. Fox, et al.,[10,11] have found colloidal or otherwise dispersed organic detritus in mid- and deep-ocean waters as well as nearshore waters. It follows that the persistence of this material in the water column is determined by the rate of bacterial decomposition and mineralization rather than flocculation and settling.

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[5] "Recent Sediments of the Gulf of Paria," by Tj. van Andel and H. Postma, Reports of the Orinoco Shelf Expedition, Vol. 1, North Holland Pub. Co., Amsterdam, 1954.

[6] "Chemistry of Sedimentation," by U. G. Whitehouse, in Study of Nearshore Recent Sediments and Their Environments, API Research Project 51, Reference 52-51, Scripps Inst. of Oceanography, 1952, pp. 23-32.

[7] "Chemistry of Sedimentation," by U. G. Whitehouse and L. M. Jeffrey, Reference 53-43, Scripps Inst. of Oceanography, 1953, pp. 31-38.

[8] "Diagenetic Modification of Clay Mineral Types in Artificial Sea Water," by U. G. Whitehouse and R. S. McCarter, Clays and Clay Minerals, Natl. Acad. of Sciences-Natl. Research Council, Washington, D. C., Publication 566-1958, pp. 81-119 (Includes reference to 6 and 7 above).

[9] "Differential Settling Tendencies of Clay Minerals in Saline Waters," by U. G. Whitehouse, L. M. Jeffrey, and J. D. Debbrecht, Proceedings, Seventh Natl. Conf. on Clays and Clay Minerals, Pergamon Press, 1960, pp. 1-79.

[10] "Marine Leptopel, its Recovery, Measurement, and Distribution," by D. L. Fox, J. D. Isaacs, and E. F. Corcoran, Journal of Marine Research, Vol. 11, No. 1, 1952, pp. 29-46.

[11] "Particulate Organic Detritus," by D. L. Fox, Geol. Soc. of America. Memoir 67, Vol. 1, 1957, pp. 383-390.

The previously cited works are not directly applicable to waste disposal. Nevertheless, it is reasonable to expect that at least a portion of the effluent solids will respond to various environmental factors in manners similar to those of natural compounds. Certainly, differential flocculation phenomena must be considered in evaluating the rate and location of deposition of effluent suspended solids and their constituents. The converse of this proposition, that dispersed, unflocculated solids will be transported for greater distances may have a similar significance.

One of the conclusions derived from the marine studies of the disappearance rates of coliform bacteria from Hyperion discharges was that primary effluent was more amenable to final treatment by the ocean than was secondary effluent.[2] Two approaches have been used in attempting to verify these results under controlled laboratory conditions.

In the first, various sea water dilutions of the effluents were placed in large cylindrical columns, and tests were made over a period of time of coliform densitites at different depths. It was hoped that the tests would indicate a rate of sedimentation of the bacteria and a consequent reduction of bacterial populations at the surface. No such reduction was found, probably because of convection currents being set up in the column by small temperature gradients in the room and because of the motility of the bacteria themselves.

The second series of tests involved centrifuging various sea water dilutions. (The writer is indebted to K. J. Mysels of the University of Southern California for suggesting this technique). The tests were made at room temperature with the centrifuge operating at 1000 g's for 30 min. Qualitatively, primary effluent mixtures showed greater reductions of coliforms than did secondary, thus confirming the receiving water studies. It was also found that greater reductions were obtained with higher dilutions; this conforms with the work of Whitehouse, et al., mentioned earlier.

It was possible to determine the suspended solids reduction with assurance only for the 20-to-1 dilution of primary effluent since the absolute solids concentrations in the other dilutions were too small to be accurately measured with standard methods. The solids reduction was about two-thirds that of the bacterial reduction.

We now have three converging sets of data, the marine studies, the centrifuging experiments, and the settling tube investigations reported by Mr. McLaughlin. The differential settling characteristics of various sewage effluents are clearly demonstrated, particularly when effects of flocculation of the effluent solids in sea water are included.

Associated with the utility of centrifuging of sea water, effluent mixtures are also indicated. However, it is obvious that this technique requires a good deal of experimental and analytical study before quantitative results can be expected that will have the same reliability as the very expensive receiving water studies. In any event, it appears desirable to make laboratory tests of settling characteristics of effluent solids which may form bases of design for sewage treatment plants. Such experiments may well be designed to include the flocculating or other effects of the actual receiving water, be they fresh water or salt water.

EARLY HISTORY OF HYDROMETRY IN THE UNITED STATES[a]

Discussion by Murray Blanchard, Arthur H. Frazier, and J. C. Stevens

MURRAY BLANCHARD,[73] F. ASCE.—The author has compiled a very interesting history of hydrometry. Under the heading Great Lakes Investigations he has given a complete resume of the equipment and methods developed and used by the engineers of the United States Lake Survey on this work.

The writer had an opportunity to make use of those pioneer developments on the following hydraulic investigations of the Great Lakes:

1. Discharge from Lake Superior of the St. Marys River at Sault Ste. Marie Mich., for the United States Lake Survey.
2. Discharge from Lake Huron of the St. Clair River at Port Huron, Mich., for the United States Lake Survey.
3. Discharge of the Detroit River between Lakes St. Clair and Erie at Detroit, Mich., for the United States Lake Survey.
4. Discharge of the St. Lawrence River at the foot of Lake St. Francis near Valley Field Ontario, Canada, for the J. G. White Company of New York in connection with Water Power development investigations.
5. Discharge of the Chicago Sanitary District's Main Channel from Lake Michigan at Lemont, Ill., for The Sanitary District of Chicago in connection with United States Lake Level Litigation.[74]

To the hydrometric investigations of the United States Lake Survey referred to by the author there should be added discharge measurements of the outflow of Lake Superior made in 1901 and 1902 by H. F. Johnson and W. Edward Wilson from the International Bridge at Sault Ste. Marie, Mich.

ARTHUR H. FRAZIER.[75]—One finds it hard to realize that this paper on American engineering history has been written by a person who has lived in America for only twelve years. No doubt most of the native-born Americans who read it found much information therein that was entirely new to them. It is accordingly with a humble spirit that this discussion is offered.

In only one detail do I find reason to disagree with data contained in the paper. The author states that the first discharge measurement made in the State of Maine was made by G. H. Hamlin of Maine State College who measured the Penobscot River in 1884, whereas George L. Vose of the Massachusetts Institute of Technology wrote[76] in 1885 that:

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[a] January, 1960, by Steponas Kolupaila.

[73] Retired Civ. Engr., Libertyville, Ill.

[74] M. Blanchard, Hydraulics of Chicago Sanitary District's Main Channel, Journal of the Western Soc. of Engrs., Vol. 25, No. 13, Sept. 5, 1920, pp. 471-524. Discussion: Vol. 26, No. 8, August, 1921, pp. 300-303, Chicago.

[75] Retired member of the U. S. Geodetic Survey.

[76] "A Sketch of the Life and Works of Loammi Baldwin," by George L. Vose, Boston, 1885.

"In 1835, several prominent gentlemen in Maine became interested in the development of the water power of the Androscoggin River at Brunswick; and Mr. Baldwin was employed to make the necessary measurements and computations . . . . From the several gaugings, 4,000 cubic feet per second were reckoned safe as the discharge which might be relied upon as constant through the year for mill power; and with forty feet fall in the river at this place, Mr. Baldwin concluded that the Androscoggin at Brunswick offered a power unsurpassed by any in New England at that time occupied."

As a matter of fact, it seems quite unlikely that even Baldwin's measurement was the first one made in Maine. A contemporary, and no doubt a close friend of his, Ithamar A. Beard, made surveys for the erection of a large cotton factory in Saco, Maine, in 1831. Not long afterward Beard advertised (in the Lowell Courier during October, 1840) that he, as a civil engineer, offered his services "in surveying land . . . , *in measuring and computing the quantity of water supplied by streams* . . . , and generally, in any business pertaining to engineering." His records of streamflow measurements, and those of similar engineers of his time, would provide interesting comparisons with present-day data if they could be found. It is more than likely that many of Beard's measurements would have been made earlier than any presently on record for the states in which he lived (Maine, New Hampshire, and Massachusetts).

In writing a history such as this, one is always faced with the problem of what items might be omitted for the sake of brevity. That circumstance always provides an almost unlimited area for discussion purposes. Advantage will be taken thereof in the comments that follow.

Perhaps "Jean" Nicolas Nicollet was singled out among the explorers of the Upper Mississippi River because of the magnificent map he made of the area at that early data (1836-1839). He was not, however, the first government-sponsored explorer to have undertaken the job. Colonel Zebulan Pike was ordered to search for the head-waters of that river in 1805. General Cass and later (1832) Henry Rowe Schoolcraft also were sent on that mission. Of those three predecessors of Nicollet, only one, Henry Rowe Schoolcraft, actually reached Lake Itaska (the nominal "headwaters" of the Mississippi). It is a remarkable coincidence that a still earlier explorer in this area also bore the name of Jean Nocolet. In 1634 this earlier Jean Nicolet discovered Lake Michigan, and even undertook to journey up the Fox River in Wisconsin in quest of the "Father of Waters," but he did not go far, probably only as far as Lake Winnebago, and then turned back, only a day's journey away from his goal. For the sake of the record, however, a question might be raised regarding the given name attributed to "Jean" Nicholas Nicollet by the author. Most American sources have translated his first name as "Joseph," rather than "Jean."

From a historical viewpoint it would seem that a report made as of April 4, 1808 on canals by Albert Gallatin, Secretary of the Treasury, might well have been mentioned in the section on such projects, also in the second appendix. That report, which reviewed the status of all existing public roads and canals under construction (or completed) as of the date it was written, is available.[77]

[77] The Historical Register of the United States, Part I for 1814, edited by T. H. Palmer, and published by G. Palmer, Philadelphia, 1814.

And finally, there seems reason to believe that William Hammond Hall, California first state engineer, might have deserved a little more attention. He is mentioned in the section pertaining to the "Progress of Hydrometry in the West," but possibly with too little explanation regarding his fine engineering accomplishments. Irrigation received its first recognition on a state-wide basis in the state of California, and Mr. Hall was the first man to have had

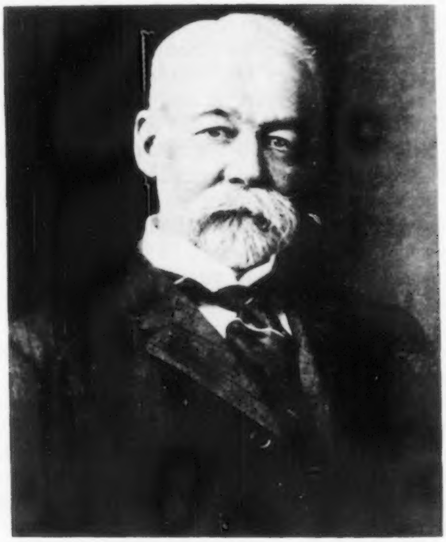

FIG. 62.—WILLIAM HAMMOND HALL

charge of that project. So skillfully did he undertake such work that Captain C. E. Dutton reported.[78] "In this State (California), irrigation has made greater advances than in any other."

As a pioneer in this field, it was necessary for Mr. Hall to develop stream-gaging methods and equipment as well as irrigation procedures. No doubt he was the nation's foremost authority on those subjects during his period of administration. An example of his design work on stream-gaging equipment is the design of the first American cable-way for making streamflow measurements. He also improved the design of two current meters, the first of which his assistant, C. E. Grunsky described as a "Henry type." The second current meter on which he made improvements was of the Small-Haskell type. He loaned several of this second group to the U. S. Geological Survey in later years for conducting its work in Colorado and Nevada. Surely, William Hammond Hall deserves having his picture in this "Hall of Fame" as much as his assistant, C. E. Grunsky.

J. C. STEVENS,[79] F. ASCE.—In the writer's opinion, this paper is the hydraulics classic of the 20th century. It is not a treatise on hydrometry and

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78 First Annual Report of the U. S. Irrigation Survey, (later converted into the U. S. Geological Survey).

79 Cons. Engr. and Partner, Leupold and Stevens Instrument Co., Portland, Ore.

hydraulics. It is a treatise on men who made those sciences click in the United States since the beginning of its history. And this is not all. Mr. Kolupaila is preparing a worldwide bibliography of Hydrometry from the beginning of written history, some 6,000 years ago, to the present. Only one with his broad knowledge of hydraulics and hydrometry and his versatility in languages could successfully undertake such a momentous task.

As an example of the author's thoroughness, the writer is aware of the following incident. C. S. Jarvis[80] has stated: "Some of the sub-normal Nile flood stages were increased arbitrarily by order of unscrupulous rulers." In Mr. Kolupaila's bibliography this nefarious desecration is pinned down thus: "The rulers of Egypt established land taxes called Kharag, corresponding to the top of the Nile flood in that year. It was in the best interests of the rulers to have the Wafa (When the Nile reached the 16-cubit (28-foot) stage the sheikh, a guardian of the nilometer after the Arabs conquered Egypt, had to proclaim the "Wafa," a beginning of the annual fast.) proclaimed earlier and the top level read higher in order to entitle them to start the tax collection earlier and to increase the assessment. The scale was doctored for that purpose by order of the Caliph Omar: the cubits between 16 and 22 marks were shortened by one-half. This fraudulent alteration resulted in riots of the population and spoiled the value of the longest record of river observation extending over 1,300 years."

The writer is wondering whether the relation of forests to water supplies could properly be included in the category of hydrometry. At the turn of this century the forest-water supply relationship was one of the major topics for the lecture platform and periodicals. The reason for this lies in the fact that this was the time when the nation was debating, in Congress and the press, the proposal that the government should acquire large areas of forest lands and administer them as National Forest Reserves. The conservationists were urging their acquisition, but there was one hurdle congress had to make before such a course could be legalized.

Congress had authority over river navigation, but unless it could be shown that forests were a substantial aid to navigation congress was legally powerless to acquire and administer forest reserves. The ardent conservationists therefore propagandized the nation with all the weapons at the command.

Two examples will be cited:

1. Gifford Pinchot, probably the greatest exponent of forestry the world ever knew states:[81] "The connection between forests and rivers is like that between father and son. No forests, no rivers." From this it would appear that forestry is a substantial adjunct to hydrometry.

2. In the crescendo of direful prophecies, this one from the scrap book of September, 1908, wins the blue ribbon: "When our forests are gone, the streams will dry up, the rivers will cease to run, rain will fall no more and America will be a desert."

One might add "and hydrometry will be but a memory." One can imagine Teddy Roosevelt saying to his tennis cabinet: "Go to it, boys, the end justifies the means. Pull out all the stops. Make America forest conscious. Never mind the facts. We've got to create forest reserves."

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80 "Flood State Records for the Nile River," by C. S. Jarvis, Transactions, ASCE, Vol. 101, 1936, p. 1026.

81 "The Fight for Conservation," 1909, p. 53.

One of the first voices raised against this flood of falsification was a lengthy and scholarly paper[82] by General H. M. Chittenden of the U. S. Army Engineers.

The writer has compiled[83] all known long time records of stream flow which are analyzed with reference to the forest cover on their drainage areas.

Thus, out of a welter of conjecture and falsification, grew one of our most valuable national assets. National forests now cover one-third of our land area. With the sustained yield system now in force not alone by federal but also by many private owners, we are assured a perpetual source of lumber and as a corollary unrivalled recreation facilities, while soil erosion is kept to a minimum in those areas. The end did justify the means.

One tie the forests have with hydrometry is the fact that some forested areas are actually being cut off to increase the water supply with the knowledge and acquiescence of forest management. A forest is a great dissipater of water by evaporation from limbs and leaves and by the extraction of water from the soil in growth processes. It is obvious, therefore, if the forest is removed more water will be available for storage and use. It appears, therefore, that forestry in its many phases may definitely be akin to hydrometry.

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[82] Transactions, ASCE, Vol. LXII, March, 1909, "Forests and Reservoirs in Relation to Stream Flow with Particular Reference to Navigable Rivers."

[83] Journal of the Assoc. of Engrg. Socs., Boston, July, 1913, "Forests and Their Effect on Climate, Water Supply and Soil."

GENERALIZED DISTRIBUTION NETWORK HEAD LOSS CHARACTERISTICS[a]

Discussion by Paul C. Constant, Jr., Marcel Bitoun, Claud C. Lomax, Joseph W. Maier and Thomas C. Miller, and J. M. Robertson

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PAUL C. CONSTANT, JR.[4]—The author describes a mathematical method by which pressure conditions of an existing water distribution system can be obtained easily for different ratios of Q_p/Q_d. Before the method can be used, there must be available at least one set of measured data for two different Q_p/Q_d ratios. Also, the various head losses are calculated under the assumption that they are a result of Q_p/Q_d ratios where the loads are proportionately changed and all other system parameters remain unchanged.

Although the restrictions imposed seem quite severe, the author's method for calculating different head loss conditions is useful for municipality distribution operations for a "closed" system, that is, the operating end of the business. The author very clearly indicates the usefulness of his method for obtaining intermediate pressure conditions. However, he does not stress that application of his method of calculating head losses is mainly for the operations end of distribution problems. An example where the author's method may be used effectively is in the determination of pressure gradients in a system for a "drought" year.

Random checks on the author's calculations for n and ϕ were made using the data from Table 1. An error, Δn, no greater than 5% of n (n = 2.46) was found. Further calculations were made to determine the error, $\Delta\phi$, in ϕ (ϕ = 0.0344) due to Δn in n. This error was no greater than 2%. These calculations indicate that the author's method is within the limits of engineering accuracy. In fact, the errors are trivial because (a) estimates are made on demands, (b) loads are grouped, and (c) of error in meter readings.

The approximate value of the relative error in $\left(\frac{\sum h}{{Q_d}^{1.85}}\right)$ may be determined easily be using the general formula for errors,

$$E_r = \frac{\Delta N}{N} \qquad (2)$$

where

$$N = \frac{\partial N}{\partial \mu_1} \Delta\mu_1 + \frac{\partial N}{\partial \mu_2} \Delta\mu_2 \ldots + \frac{\partial N}{\partial \mu_k} \Delta\mu_k \qquad (3)$$

in which E_r represents the relative error; N is the function of several independent quantities; μ_i denotes the parameters (i = 1, 2, 3, . . . , k); and $\Delta\mu_i$

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[a] January, 1960, by M. B. McPherson.

[4] Engrg. Dept., Midwest Research Inst., Kansas City, Mi.

represents errors in the parameters. The general formula, Eq. 2, can be applied to the author's equation,

$$\frac{\sum h}{Q_d^{1.85}} = \phi \left(\frac{Q_p}{Q_d}\right)^n \qquad (4)$$

to give the approximate relative error in the calculation of the function $\left(\frac{\sum h}{Q_d^{1.85}}\right)$,

$$E_r = \frac{Q_d^{1.85}}{\sum h} \left\{ \left(\frac{Q_p}{Q_d}\right)^n \Delta\phi + \left[\phi \left(\frac{Q_p}{Q_d}\right)^n \text{Log}_e \left(\frac{Q_p}{Q_d}\right)\right] \Delta n \right\} \qquad (5)$$

in which Q_p and Q_d are treated as constants, and Δn and $\Delta\phi$ are the errors in the calculated values of n and ϕ, respectively.

The approximate relative error in $\frac{\sum h}{Q_d^{1.85}}$ was found to be insignificant by using Eq. 5, and selecting $\Delta\phi$ and Δn to be the errors previously calculated.

There is an error in the paper under the heading PROPORTIONAL LOAD CHARACTERISTICS WITH EQUALIZING STORAGE where the author solves for n. The number 0.67 should read 1.67.

The author could enlarge upon the analysis of a particular distribution system with the investigation of flows in the various pipelines. This is a natural consequence of having the head losses. The flows can be calculated simply by means of the Hazen-Williams hydraulic formula (constant density) in the form,

$$Q = \left(\frac{h}{A\ k_p}\right)^{1/1.85} \qquad (6)$$

in which Q is the flow rate; k_p is the head-loss coefficient of the pipeline; A denotes the arbitrary constant to account for flow units; and h represents the head loss.

The method of calculations described in the paper are of little value to the person concerned with improvements of a particular distribution system; such as, (a) determination of the best location for pipelines, (b) comparing alternate arrangements of systems planned for construction, (c) optimizing a system so as to minimize the number of sources required, or (d) selection of pipeline diameters for the best combination of economy and performance. These problems, along with others, seem to be in the majority for consultants in the field of water and gas pipeline distribution analyses. This, in part, was implied by the author's statement, "It must be realized that these values of ϕ and n would be changed if either the load or the piping in the network of Fig. 5 were modified in any way, or if m was changed."

MARCEL BITOUN,[5] M. ASCE.—Publication, by the author, of the results of the experiment analysis of large, actual networks is very valuable. Such data

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[5] Chf., Design Branch, Div. of Flood Control, Pennsylvania Dept. of Forests and Waters, Harrisburg, Pa.

are, unfortunately, too scarce. The author is also to be complimented for having put in evidence definite characteristics of distribution systems with mixed types of loading (residential and industrial).

A lengthy part of the paper has been devoted to the justification of an hydraulic property of proportionally loaded networks. Three distribution systems of increasing complexity were analyzed and used to empirically justify the fact that "with proportional loads the percentage distribution of flow in individual pipes in a balanced network is constant irrespective of the magnitude of the total demand." The author expressed this as a "contention" in his conclusions.

This property has been used by the writer and, it is presumed by other hydraulic engineers for the analysis of balanced networks in which the primary interest was in the distribution of discharges rather than the determination of head losses. Actual flow values were replaced by proportional figures, the largest being assumed to be 100 arbitrary units. Sometimes cumbersome figures were thus eliminated. The writer wishes to submit here a demonstration of the validity of the hydraulic property empirically deduced by Mr. McPherson.

Consider a distribution system N without equalization storage governed by boundary conditions expressed by given ingoing and outgoing discharges. In this system, let F be one balanced regime of flow, characterized by a set of ingoing and outgoing discharges, and the corresponding values of flows in the individual pipes that compose the system.

We have, for any junction of this system,

$$\sum_{i=1}^{r} Q_i = 0 \qquad (7)$$

in which r is the number of pipes connected at the junction.

In any loop of the system, we have

$$\sum_{i=1}^{n} K_i \, Q_i^{m} = 0 \qquad (8)$$

in which n is the number of pipes in the loop.

Let us multiply all boundary discharges by a factor q ($q \neq 0$), and let F' be the new regime of flow in the system.

In any loop Eq. 7 can be multiplied by q and yield

$$\sum_{i=1}^{r} q \, Q_i = 0 \qquad (9)$$

If

$$Q'_i = q \, Q_i$$

then

$$\sum_{i=1}^{r} Q'_i = 0 \qquad (10)$$

Eq. 8 multiplied by q^m yields

$$\sum_{i=1}^{n} K_i \; q^m \; Q_i^m = 0 \quad \ldots\ldots\ldots\ldots\ldots (11)$$

If

$$Q'_i = q \; Q_i$$

then

$$\sum_{i=1}^{n} K_i \; Q_i'^m = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Eqs. 10 and 12, valid for any junction and any loop respectively, show that;

1) the regime F' is hydraulically balanced,
2) the flow for F' in every individual pipe is equal to the flow for F in the same pipe multiplied by the factor q.

Moreover, this flow distribution is the only one compatible with the given boundary discharges. Demonstration of the unicity of the flow distribution in a system where boundary discharges are given was furnished by Angles d'Auriac.[12]

Therefore, since the regime F', in which all discharges are those characterizing F multiplied by a constant factor, is a solution, and since this solution is the only flow distribution in the network corresponding to the proportional load, we have shown that the flow in each individual pipe is a constant fraction of the total load, which confirms Mr. McPherson's findings.

The number of computations made on the analyzer for the construction of Fig. 4 could therefore have been considerably reduced. A single analysis for each network would have been sufficient to provide one basic flow distribution.

The second comment which is offered here refers to ϕ and n. For Mr. McPherson's Eq. 2,

$$\frac{\sum h}{Q_d^{\,m}} = \phi \left(Q_p/Q_d\right)^n ,$$

it was stated that for $\frac{Q_p}{Q_d} = 1, \; \phi = K.$

However, two field tests were conducted for each location. In fact, only one test is necessary. The parameter ϕ is actually the hydraulic resistance of the system, and can be computed from the physical characteristics of the pipes which compose it, provided, of course, that these characteristics (C, L, D) are known. A single field test will furnish the second equation necessary for the determination of n.

The last comment refers to the relative influence of non-proportional variations of the industrial loading and of the residential loading in the same network. It is felt by the writer, although this is not a certainty, that deviations

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[12] "A propos de l'unicité de solution dans les problèmes de réseaux maillés," (About the unicity of the solution to network problems), La Houille Blanche, No. 3, May-June, 1947, Grenoble, France, p. 209.

from a proportional loading within a certain range will not result in important differences in the distribution of flow in the system. This would be due to a "dampening" of the difference throughout the system. Comments of the author based on his experience on this point and especially on results of computations made on the analyzer might prove conclusive.

CLAUD C. LOMAX,[6] M. ASCE.—The author is fortunate in having had an opportunity to compare actual flow conditions in a pipe network with the results obtained on a McIlroy Analyzer. The writer has not had that opportunity and must present his comment in the light of this lack of experience.

From a theoretical standpoint, the technique of proportional load hydraulics can be firmly supported. The comparisons that the author makes between the McIlroy Analyzer results and the actual flow distributions and the pressures, show some discrepancies. The differences may be attributed, in part, to the following: metering limitations, Fluistor selection limitations, and mistakes that may have occurred in the McIlroy Analyzer solution. The conformity of the comparisons is effected by the reliability of the field measurements of flow distribution, pressure losses, and data such as pipe sizes, lengths, and friction factors. The writer is of the opinion that the results presented in this paper are well within the range of discrepancy to be expected.

In the light of the writer's experience, which has been primarily with distribution networks for small western cities, a word of caution on general application of proportional load hydraulics is in order. In the smaller cities the preparation of a skeletonized or arterial network is usually not feasible, because there may be only one run of large pipe from a pumping station or reservoir, and the remainder of the network is made up of 4 in., 6 in., and 8-in. sizes. The author stated, "This is the smallest distribution district in Philadelphia and the only one in which fire flows governed the design of future arterial piping requirements." For smaller cities the fire flows are usually controlling. For western areas irrigation loads are significant parts of the total system load. Consequently, it is the writer's opinion that application of proportional loads for design conditions of flow networks in small cities is not feasible.

The author has not so stated, but it is implied that the source magnitudes must be proportional as well as the load magnitudes. The smaller cities draw from both reservoir and well sources, and usually all sources are required to satisfy the maximum demand condition, whereas conditions less than maximum require pumps for reservoirs that may be scattered throughout the community.

It would be very difficult to achieve proportionality of source magnitudes in a system where reservoir water surface elevations and pump discharges are fixed. One would need additional pump capacities at each pumping station, which were proportional to the percentage contributed by that station as well as proportional to the total load increase. Fire loads, when superimposed on the maximum day demand for irrigation, domestic, and industrial usage, do not increase the load on the system proportionately since these fire loads are placed at specific hydrants selected throughout the community. The use of proportional load hydraulics in the smaller cities may be valuable, if judiciously applied, to determine the capacity of the system for flow distributions greater than the maximum day demands. A clue to the behavior of the system with

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[6] Assoc. Hydr. Engr., R. L. Albrook Hydr. Lab., Washington State Univ., Pullman, Wash.

fire loads can be obtained in this way. However, the pressures and flow distributions in the immediate vicinity of those hydrants being used may be greatly in error.

Having used the McIlroy Analyzer for a considerable number of flow distributions, the writer, and probably others having had close contact with several McIlroy Analyzer distributions, is impressed by the feeling one gets for a "healthy" network distribution system. In a well-designed system the omission of a section of pipe, other than the critical section leading to a reservoir or pumping station, usually does not seriously limit the capacity of the system. Through errors in wiring or selection of incorrect Fluistors, one accidentally sees this demonstrated on a network, in that it is not really obvious that an error has been made until a check procedure discovers this error. On other networks, which are "weak," the omission of almost any pipe in the network distribution system will reduce its capacity. Proportional load hydraulics is probably much more applicable to a "healthy" system than a "weak" one.

With very low flows, conditions may be present wherein laminar flow exists in some of the pipes. To apply proportional load hydraulics and increase the flow to the point where these flows become turbulent, is certainly unwarranted.

Referring to the author's statement on gas flows, it should be noted that gas flow distributions are related to the 1.85 power of the flow when the Fritzche or Panhandle formulas are used.

The author is to be complimented for having presented, with supporting evidence of its validity, a technique of analysis which, if judiciously applied, will serve designers of flow distribution systems. It should be obvious that one does not require a McIlroy Analyzer analysis nor a complete system analysis to initiate use of this technique. Having pressures at critical points in a distribution network and assuming the system proportionally loaded, one can then evaluate the total capacity of the system. Of course, one can start with a McIlroy network analysis as well.

JOSEPH W. MAIER[7] and THOMAS C. MILLER,[8] A.M. ASCE.—The proportional loading principle, as presented in this paper, does not seem to be applicable to gas system networks. The term "proportional load" is described as the typical design assumption that each individual consolidated load varies or fluctuates about its mean in direct proportion to the total system fluctuations. Thus the maximum and minimum hour demands of the average and maximum day would be taken as fixed multiples, not only of the annual averages for the district as a whole, but also for each individual local load. It has been the writers' experience that a consolidated load fluctuates according to the characteristics of the primary gas load of which it is composed. From this load characteristic fluctuation, it can be further concluded that a fixed multiply representing total system demand variation is apparently not applicable to individual consolidated loads in a gas distribution system.

The composition of a consolidated gas load will include gas used for one or more of the following purposes: commercial, industrial, house heating, cooking, water heating, refrigeration, and drying. A consolidated load consisting of industrial customers will not realize appreciable yearly variation, while a consolidated load composed of residential house heating customers will be fully

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[7] Asst. Engr. of Distribution Design, Philadelphia Gas Works, Philadelphia, Pa.
[8] Staff Engr., Philadelphia Gas Works, Philadelphia, Pa.

affected by temperature and wind velocity variations. It can be seen that each of the above mentioned gas usages has specific characteristics which when considered in a network analysis cannot be represented by a fixed multiply. In addition, seasonal industrial customers are supplied during the non-heating season.

To illustrate gas load fluctuation, let us use Figs. 8 and 9. Fig. 8 is a schematic flow breakdown of a winter day, in which Q_t is the total hourly sendout, Q_1 refers to the industrial customers, Q_2 indicates the residential non-heating customers, and Q_3 is the residential heating customers, while Fig. 9, in which

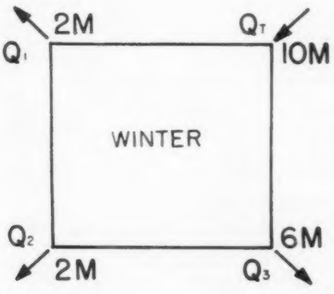

FIG. 8

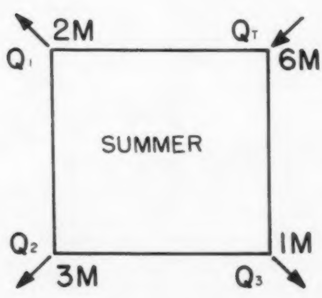

FIG. 9

Q_2 is the residential non-heat customers plus seasonal industrial customers, and Q_3 refers to the non heat (base load) of residential heating customers, represents the same network on a summer day. Although the summer hour sendout is 60% of the maximum winter hour sendout, the consolidated loads of the summer day (Q_1, Q_2, Q_3) are not 60% fixed multiplies of the corresponding maximum winter loads.

J. M. ROBERTSON,[9] F. ASCE.—It seems desirable to point out that the solution of pipe network flow and head loss problems with a digital computer

[9] Prof. of Theoretical and Applied Mechanics, Univ. of Illinois, Urbana, Ill.

after the Hardy Cross technique has been improperly termed relaxation. Although many authors use the term relaxation so as to include the iteration process employed in the Cross approach, there is a distinction that should be observed. The error in terminology is not surprising since the introduction of the iteration technique of obtaining solutions to such problems by H. Cross[10] was but a by-product of his work in structural analysis, where he did introduce the technique (as noted by G. E. Forsythe,[11] a relaxation technique was probably used as early as 1823 by K. F. Gauss for solving the simultaneous equations encountered in surveying) later named relaxation by R. V. Southwell.

The pipe network analysis technique is sometimes termed the Cross-Doland method, since it was at the suggestion of J. J. Doland[12] that Cross considered the application of the methods of numerical successive approximation to network problems. As noted recently by T. G. Chapman,[13] pipe networks may be solved either by iteration or by relaxation methods. The relaxation method is primarily limited to paper and pencil type calculations, to which it is ideally suited and for which it was originally devised, as so clearly stated by G. E. Forsythe.[14]

In spite of the fact that relaxation techniques are applicable to various types of fluid flow problems, which we may now attempt to solve with the help of high speed digital computers, solutions of such problems on digital computers are rarely, if ever, relaxation solutions. They are merely iteration solutions—the two terms are not synonymous. This is because relaxation employs judgment and reasoning, if not intuition, at every step. These are beyond the usual mentality which is programmed into a digital computer. Such a machine can merely plug along, at a very high rate of speed of course, making one simple calculation after another in accord with a rigid prescribed program. Relaxation techniques, on the other hand, accelerate the iterative convergence of solutions through various strategems. Thus, computation is first applied to regions where the divergence from a proper local result is greatest. In addition, such techniques as block and over relaxation are employed where suitable. Since judgment is required in deciding when and what technique to employ this can hardly be programmed into a dumb computer. Or, if it can be, the program is just too complicated, too involved. Quite probably nothing is impossible on a computer, but it is better to do a simple computation many times than to try a complicated one that can achieve the same computation in one cycle. Thus, as Southwell himself has said, it is best to set a computing machine to work with an iterative process. It is the present writer's understanding that such is indeed the case with distribution network analyses.

The question of the value of the exponent m in the power-law (sometimes incorrectly termed exponential law) head-loss relation is raised by the author. Although this exponent may assume values between 1.75 and 2.00, network

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10 "Analysis of Flow in Networks of Conduits or Conductors," by H. Cross, Univ. of Illinois, Engrg. Experiment Sta. Bulletin No. 286, November 13, 1936.

11 "Gauss to Gerling on Relaxation," by G. E. Forsythe, (translation and commentary on a letter by Gauss dated December 25, 1823) Mathematical Tables and Other Aids to Computation, Natl. Research Council, Vol. V, 1951, pp. 255-258.

12 "Simplified Analysis of Flow in Water Distribution Systems," by J. J. Doland, Engineering News-Record, Vol. 117, October 10, 1936, pp. 475-477.

13 "Relaxation Methods for Pipe Networks," by T. G. Chapman, Civil Engineering and Public Works Review, Vol. 1, November, 1956, pp. 1247-1248.

14 "What are Relaxation Methods," by G. E. Forsythe, Chapter 17 of Modern Mathematics for Engineers, Ed E. F. Beckenbach, McGraw-Hill Book Co., 1956.

analysis progresses best if a single value is used for any given analysis. Certainly the author's proportional load hydraulics method is not applicable if several pipes of different m values occur in the system. Network solutions techniques may not be limited to fixed-m (versus variable or several m) systems, but the exigencies of the approaches to the solution, such as are being discussed, often so limit them. Certainly any paper and pencil analysis (plus slide rule or desk calculator and table of powers of numbers) whether merely iterative or relaxation, can easily employ different m values for each pipe.[13] The McIlroy network analyzer may be used with m other than 1.85. Different values could even be employed for different pipes, but this is rather troublesome. Digital computer iteration solutions have been employed for m values of 2.00 and less, but with the same value for all pipes in any given network. L. R. Henry and R. B. Peritz[15] note, however, that the use of non-integer values of the exponent tied up a large part of an IBM 650 machine for the process of raising the flow rate to this power. A digital computer cannot do this nearly as simply as one can with an analog computer (such as a slide rule, for instance). This extra effort reduces the number of pipes that can be handed over that number which can be handed when m is 2.

The programming complexities of using different m values for the several pipes of a system are obviously much greater, since a special Q^m subroutine would be needed for each value of m to be employed. Either an extensive table of such results for a limited set of m values would be needed or some analog auxilliary must be developed to which the digital computer may refer quickly when Q^m is needed. The first method is probably impractical due to the access time required, while the second is not available. Since the range of flows in a given pipe is usually small, there seems to be no need to consider pipe lines of variable m. It would thus appear that for some time to come our solutions of network systems is limited to a few fixed values of the head loss exponent. Finally, of course, there remains the practical question of whether the exponent can be determined accurately enough to justify considering anything more than an average value.

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[15] "Computer Evaluates Design of Gas Gathering System," by L. R. Henry and R. B. Peritz, Oil and Gas Journal, Vol. 57, No. 17, April, 20, 1959. Also "Evaluation of Natural-Gas System Design with an Electronic Computer," ASME Paper No. 58-Pet-28 (for a condensed version see Mechanical Engineering, June 1959, pp. 58-59.)

THE FOURTH ROOT n-f DIAGRAM[a]

Discussion by Gerald Lacey and N. Rajaratnam

GERALD LACEY.[11]—The author in writing his paper has, manifestly, two objects in mind: first the presentation of a new friction-factor diagram for the use of practical engineers, and secondly that of challenging the validity of logarithmic flow formulas.

Provided that the engineer makes good use of all the information at his disposal it would seem that it makes little difference whether he uses the Moody Diagram which is effectively that of Colebrook and White, or the Blench diagram. The author suggests that the latter should be employed and judged by results. There is little doubt that the author's diagram can be usefully employed but such use can neither prove not disprove the validity of the logarithmic flow formulas which, at present, hold the field.

Those who have made a study of recent publications on the theory of flow in pipes might well be inclined to conclude that, so far as closed conduits are concerned, theory is now approaching finality. The use by Colebrook of the title "Smooth and Rough Pipe Laws" supports this view. It is these laws, so-called, in their logarithmic form which the author believes to lack a completely logical foundation or verification in the range of practical data. Such "proofs," or more properly physical explanations, as have been put forward in the academic world, he regards as patently illogical and mathematically obscured. The author puts forward as an alternative his Fourth Root Theory based on the postulate of a universal flow formula for all boundary types which, it would appear, would embrace open channels with moving boundaries as well as pipes with rigid boundaries. The author's diagram, practical although it may be, is essentially a vehicle for the presentation of the author's theories of flow. They do not lack originality.

The author draws attention to the fact that Prandtl was careful to explain that his proof of a logarithmic velocity distribution did not apply to pipes but it would have been of value if the author had stated when, if ever, the proof was valid—in the case of an open channel for example. The point is relevant inasmuch as the author, is support of his fourth root theory as applied to pipes, has no hesitation in making use of observations on open irrigation channels, transporting sediment, and with moving boundaries.

So far as rigid boundaries are concerned it is well established that with full turbulence and low values of the relative roughness the logarithmic equation can be replaced over a large range by the Manning formula with little error.

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[a] January, 1960, by T. Blench.
[11] Cons., Sir M. MacDonald and Partners, London, England.

The contention of the author that it is "believed" that Manning's n is slightly dependent on the size of pipe is debatable.

When the author refers to "absolute roughness," and the well known fact in irrigation canal practice that Manning's n for exact design has to be reduced for large channels of the same apparent boundary roughness, he stands on firm ground. The writer more than 30 yr ago (about 1930) was responsible for the formula on which the author relies[12]

$$V = \frac{1}{N_a} R^{\frac{3}{4}} S^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

(this equation being in metric units), and commented that

> "the coefficient was one of absolute rugosity, equal to that of Kutter and Manning . . . when the hydraulic mean depth was 1 metre, but remaining a true constant so long as . . . the size of the silt particle was constant. The need to vary the rugosity coefficient with the size of the channel vanished."

This equation however applies to open channels with moving boundaries, and possibly a very different velocity distribution from that in a channel with a rigid boundary.

There is no doubt that the author's diagram has the merits of great simplicity. Whether the practical engineer should be weaned from his predilection for the Manning equation when dealing with pipe flow and should use the fourth root equation instead is an open question. The use of the fourth root throughout may appeal to a tidy mind.

When the author states that he is content to treat his value of "e" as a code number "recognized intuitively as an observer recognizes faces," he will find many who will agree with him. The difficulty in measuring roughness quantitatively is a stumbling block and in this matter judgment and experience are of value. This may be some comfort to the practical engineer but he would like some assurance as to whether he should use the author's "e" or the more usual "k."

N. RAJARATNAM.[13]--Blench is to be highly commended for presenting a new method of solution for the problems of flow resistance in pipes.

The writer wants to make a few observations on the projected universality of the fourth root n-f diagram. Firstly, the author seems to believe that the Blasius law, given by the equation

$$f = \frac{0.316}{R^{1/4}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

describes the turbulent flow resistance in smooth pipes even for values of the Reynolds number higher than 10^5. But it is widely known that for values of the Reynolds number higher than 10^5 the flow resistance is better approximated by the Prandtl-Karman equation

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12 Proceedings, Inst., Civ. Engrs., Vol. 229, pp. 372, 373.

13 "Research Fellow, Civ. and Hydr., Indian Inst. of Science, Bangalore, India.

$$\frac{1}{\sqrt{f}} = 2 \log \; R \sqrt{f} \; -0.8 \quad \ldots\ldots\ldots\ldots\ldots (9)$$

In fact, even in the lower range, Eq. 9 agrees with Eq. 8 quite closely[14] so that when a single law of resistance is desired, it is Eq. 9 that is normally adopted.

Secondly, regarding the universal adaptability of the fourth root n-f equation, the author has suggested that if, in the equation

$$\frac{1}{\sqrt{f}} = 2.0 \left(\frac{d}{x}\right)^{1/4} \quad \ldots\ldots\ldots\ldots\ldots (10)$$

the thickness of the laminar sublayer (which is believed to exist near the smooth boundary for turbulent flow) be substituted for x the Blasius formula will be obtained. The thickness of the laminar sublayer is given[15] by the equation

$$\frac{\delta \, u_*}{\upsilon} = 11.6 \quad \ldots\ldots\ldots\ldots\ldots (11)$$

in which δ is the thickness of the laminar sublayer, u_* is the shear velocity, and υ is the kinematic viscosity of the fluid.

Substituting

$$u_* = V \sqrt{\frac{f}{8}} \quad \ldots\ldots\ldots\ldots\ldots (12)$$

in which V is the mean velocity of flow in the Eq. 11.

$$\frac{d}{\delta} = \frac{R\sqrt{f}}{11.6\sqrt{8}} \quad \ldots\ldots\ldots\ldots\ldots (13)$$

$$\alpha \; R \sqrt{f} \quad \ldots\ldots\ldots\ldots\ldots (14)$$

Substituting Eq. 13 in Eq. 11

$$R^{1/4} \; f^{5/8} = \text{constant} \quad \ldots\ldots\ldots\ldots\ldots (15)$$

as against $R^{1/4}$ f = constant which is the Blasius law.

The derivation[16] of the equation

$$\frac{1}{\sqrt{f}} = 1.75 \left(\frac{d}{e}\right)^{1/4} \quad \ldots\ldots\ldots\ldots\ldots (16)$$

by equating the values of f for Mannings, Nikuradse and Fourth Root formulas at $\frac{d}{e}$ = 150 is not clear. Further, the modification of Eq. 16 into the equation

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14 "Boundary Layer Theory," by H. Schlichting, pp. 402, 413.

15 "Elementary Mechanics of Fluids," by H. Rouse, a) p. 194 b) p. 207.

16 "Unification of Flow Formulas," by T. Blench, Proceedings, I.A.H.R., Vol. II, Lisbon, 1957.

$$\frac{1}{\sqrt{f}} = 2.0 \left(\frac{d}{e}\right)^{1/4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

seems to rob e of any possible physical significance. Eq. 17 at the outset looks like one of the many exponential formulae proposed at different times. For example, in recent times, R. M. Advani[17] has proposed a formula that can be shown to be

$$\frac{1}{\sqrt{f}} = 2.33 \left(\frac{d}{K_s}\right)^{1/6} \quad \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

in which K_s is (Colebrook's) equivalent sand roughness.

Based on Eq. 18, it can be shown that

$$K_s = (31.7\ n)^6 \text{ ft} \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

which agrees closely with the formula proposed by the writer for open channel flow.[18]

It is generally known that n decreases with increase in the size of the pipe as recently confirmed by the careful experiments of M. J. Webster and L. R. Metcalf.[19] But the variation in n seems to be very small. It is not clear as to how the author made n independent of the size of the pipe by the multiplication factor $d^{1/12}$. Webster and Metcalf data does not confirm this as shown below.

n (Maximum)	d (ft)	$n\,d^{1/12}$
0.0246	3.0	0.02695
0.0244	5.0	0.02790
0.0236	7.0	0.02770

The equation for the transition line to rough turbulent flow has been given as

$$f\ R^{1/4} = 0.80 \quad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

It has been shown[15b] that if K is the Nikuradse's sand roughness and δ is the thickness of the laminar sublayer, the transition starts at $\frac{\delta}{K} < 4.0$ and ends at $\frac{K}{\delta} > 6.0$. Based on this finding, the equation of the transition line in the Moody Chart has been shown to be

$$\frac{R\sqrt{f}}{d/2K} \approx 400 \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

This equation has been shown to hold good in many cases excepting a few discrepancies recently reported. Since, in this present case, e seems to give no

17 "Recent Developments in the Flow through Pipes and a New Method of Approach for Rough Turbulent Flow through Pipes," by R. M. Advani, Irrigation and Power, October, 1957.

18 "A Contribution to Turbulent Flow in Open Channels," by N. S. Govinda Rao and N. Rajaratnam, Presented at the 30th Research Committee Meeting of the C.B.I. & P., India, June, 1960.

19 "Friction Factor in Corrugated Metal Pipe," by M. J. Webster and L. R. Metcalf, Proceedings, ASCE, Vol. 85, No. HY 9, September, 1959.

direct magnitude of the roughness projections, the same line of argument may not be valid and hence the validity of Eq. 20 is doubtful.

It is agreed that the Colebrook-White formula, on which the curves in the transition range are based, does not give dependable results for all surfaces. In fact Colebrook[20] himself said that his equation represents a mean curve for surfaces of "tar-coated cast iron, wrought iron and galvanised iron." It has been claimed by Morris[21] that the concept of the "equivalent sand roughness" is not sound because Nikuradse's data on which it is based gives a rising curve in the transition region as against a falling curve predicted by the Colebrook-White equation. Further, the Colebrook-White equation does not make any provision for conduit surfaces having either a rising or a horizontal characteristic in the transition region. Thus it is clear that Moody's Chart does not give dependable values of f for all surfaces in the transition region. But this difficulty is still there in the fourth root n-f diagram also. Hence a new rational method of approach should be sought, having an eye for the physical picture of the flow. In this direction, an admirable attempt has recently been made by H. M. Morris.[21,22]

The writer suggests that in cases where the flow is of the wake-interference-type as in the case of many surfaces in open channel flow, it may be advisable to retain the concept of equivalent sand roughness until a better solution is evolved.

Regarding problem 6, which was worked out by the author, it is normally felt[18] that due to the additional effects of the free surface, the existence of secondary currents and non-uniform boundary shear, the equations of resistance for open channel flow are likely to be different from those for pipe flow. Based on the theoretical equations of Garbis H. Keulegan and the experimental results of Mr. Powell, the writer[18] has formulated a set of rational formulas for flow resistance in open channels with a new conversion formula for n and K_s and another formula for the transition line to rough turbulent flow.

Acknowledgment.—The writer is grateful to N. S. Govinda Rao for his interest in the preparation of this discussion.

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20 "Turbulent Flow in Pipes with Particular Reference to the Transition Region Between the Smooth and Rough Pipe Laws," by C. F. Colebrook, Proceedings, Inst., of Civ. Engrs., London, 1933, pp. 133-156.

21 "Design Methods for Flow in Rough Conduits," by H. M. Morris," Proceedings, ASCE, Vol. 85, No. HY 7, July, 1959.

22 "Flow in Rough Conduits," by H. M. Morris, Transactions, ASCE, Vol. 120, 1955.

CONSERVANCY DISTRICTS AS FLOOD CONTROL ORGANIZATIONS[a]

Discussion by Henry J. Tebow

HENRY J. TEBOW,[2] M. ASCE.—The author has presented a refreshing idea for developing additional local participation and partnership in solution of problems "in which a Federal interest is predominant," such as the problems of major rivers.

The conservancy district approach is an invaluable tool to the consultant who sometimes has difficulty, due to special interests and their desires, in winning acceptance of an over-all plan, which will best fit the district as a whole.

There should be rewarding work from a conservancy district operation, its consultants, and the state and federal engineers charged with coordination of planning, design, and construction. The conservancy district will make practicable the application of a "New Approach to Local Flood Problems,"[3] sorely needed to economize on flood control requirements for the nation.

The Colorado Conservancy District Act of May, 1937, specifically paved the way for the construction of the Colorado-Big Thompson Project, a Federal water project designed to divert surplus waters of the western side of the Continental Divide to the "water-short" acres of the eastern slope in Colorado. Though repayment planning for this Federal project allowed no credit for flood control, operation of the reservoirs to maximize the storage of snow-melt floods does provide flood control. The history of events for the project is as follows:

Initial Settlement and Irrigation in Area	1859
First State Survey of diversion ($25,000)	1889
First Bureau of Reclamation investigation	1904
Reservation for Reclamation use in Rocky Mountain National Park Act of Congress	1915
Consulting Engineer Report	1933
Public Works Administration Funds	1935
Conservancy District Organized	1937
Project authorized (Senate Document 80)	1937
Initial construction (replacement dam)	1938
First power produced (To Public Service Company of Colorado)	1943
First supplemental water delivered	1947
Initial year of major supply to lands	1954

a April, 1960, by Cloyde C. Chambers.

2 Genl. Engr., Design and Constr. Div., Region 7, Bur. of Reclamation, Denver, Colo.

3 "New Approach to Local Flood Problems," by Herbert D. Vogel, Proceedings, ASCE, Vol. 86, No. HY 2, January, 1960, pp. 53-63.

Completion for Supplemental Irrigation 1956
Project Completion (Final Powerplant of six) 1959
Project Data
- Supplemental irrigation for 720,000 acres.
- Power developed for pumping and sale 183,950 kw.
- Average surplus water diverted 257,000 Acre Feet.
- Average annual power from the water 3/4 billion KWH.
- Annual income to U. S. Treasury approximately $4 million.
- Cost of $159 million—all to be repaid.
- Power costs to be repaid with 3% interest.
- Project water value 1954 $22 of $41 million crop.

The Colorado Conservancy District Act of 1937 provides for the organization of conservancy districts by any District Court upon petition of a required number of property owners. The court appoints a board of directors that has the power to appoint officers, to acquire and hold property, to appropriate water, to enter into contracts, to levy taxes and assessments, to allot water, and generally to administer the affairs of the District.[4]

As a reflection of the indirect benefits accruling to the entire area included in a conservancy district, the act authorizes an assessment on all real and personal property not to exceed one-half mill during the construction period nor to exceed one mill thereafter, except in case of default or deficiency, when an additional one-half mill is permitted.[4]

The act further provides procedures for allotting water to individuals, municipalities, and irrigation districts and the collecting of special taxes for the use of such water. The board has the power to contract with the United States government for construction, when authorized by an election of property owners.[4]

The historical record of events for the Colorado-Big Thompson Project shows that the Conservancy District Act of 1937 was effective in initiating a plan of construction just about 50 yr after initial surveys were started.

Western pioneers, starting with the Union Colony and early irrigation near Greeley, Colorado (named after Horace Greeley—author of the quote, "Go west young man") overestimated the river water supply. Late summer and every year of drought periled the very settlements at the western end of the railroads. The 50 or more yr without construction on a water plan were certainly due to lack of a conservancy district law.

Native American conservatism, and perhaps difficult basic communication, can be given as the reason that 20 yr lapsed before the Miami Conservancy District idea went west from Ohio.

Not so with regard to the hydraulics of the MCD dry dams. Ivan E. Houck and the fine engineering papers on hydraulics, published by MCD, came west to the United States Bureau of Reclamation in the 1930's. The legal values of a Miami Conservancy District were slower moving westward.

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[4] As reported in the Tech. Record of Design and Const., Vol. 1 of 4, Colorado-Big Thompson Project, Bur. of Reclamation, Dept. of the Interior, 1957.

ERRATA

Journal of the Hydraulics Division

Proceedings of the American Society of Civil Engineers

April, 1960

p. 118 and p. 119. These pages should be placed immediately after p. 123 and before p. 124.

p. 123. Please insert the words

radius of 0.056 W was omitted. With W/D = 1.5 and an offset of 0.05 W
in Fig. 12, the pressure parameter for a point downstream

just prior to the words

from the P.I.

in line 4 of the last paragraph.

PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors (WW), divisions. Papers sponsored by the Department of Conditions of Practice are identified by the symbols (PP). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To locate papers in the Journals, the symbols after the paper number are followed by a numeral designating the issue of a particular Journal in which the paper appeared. For example, Paper 2270 is identified as 2270(ST9) which indicates that the paper is contained in the ninth issue of the Journal of the Structural Division during 1959.

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OCTOBER: 2189(AT4), 2190(AT4), 2191(AT4), 2192(AT4), 2193(AT4), 2194(EM4), 2195(EM4), 2196(EM4), 2197(EM4), 2198(EM4), 2199(EM4), 2200(HY10), 2201(HY10), 2202(HY10), 2203(PL3), 2204(PL3), 2205(PL3), 2206(PO5), 2207(PO5), 2208(PO5), 2209(PO5), 2210(SM5), 2211(SM5), 2212(SM5), 2213(SM5), 2214(SM5), 2215(SM5), 2216(SM5), 2217(SM5), 2218(ST8), 2219(ST8), 2220(EM4), 2221(ST8), 2222(ST8), 2223(ST8), 2224(HY10), 2225(HY10), 2226(PO5), 2227(PO5), 2228(PO5), 2229(ST8), 2230(EM4), 2231(EM4), 2232(AT4)[c], 2233(PL3)[c], 2234(EM4)[c], 2235(HY10)[c], 2236(SM5)[c], 2237(ST8)[c], 2238(PO5)[c], 2239(ST8), 2240(PL3).

NOVEMBER: 2241(HY11), 2242(HY11), 2243(HY11), 2244(HY11), 2245(HY11), 2246(SA6), 2247(SA6), 2248(SA6), 2249(SA6), 2250(SA6), 2251(SA6), 2252(SA6), 2253(SA6), 2254(SA6), 2255(SA6), 2256(ST9), 2257(ST9), 2258(ST9), 2259(ST9), 2260(HY11), 2261(ST9), 2262(ST9), 2263(HY11), 2264(ST9), 2265(HY11), 2266(SA6), 2267(SA6), 2268(SA6), 2269(HY11)[c], 2270(ST9).

DECEMBER: 2271(HY12)[c], 2272(CP2), 2273(HW4), 2274(HW4), 2275(HW4), 2276(HW4), 2277(HW4), 2278(HW4), 2279(HW4), 2280(HW4), 2281(IR4), 2282(IR4), 2283(IR4), 2284(IR4), [illegible](PO6), 2286(PO6), 2287(PO6), 2288(PO6), 2289(PO6), 2290(PO6), 2291(PO6), 2292(SM6), 2293(SM6), 2294(SM6), 2295(SM6), 2296(SM6), 2297(WW4), 2298(WW4), 2299(WW4), 2300(WW4), 2301(WW4), 2302(WW4), 2303(WW4), 2304(HW4), 2305(ST10), 2306(CP2), 2307(CP2), 2308(ST10), 2309(CP2), 2310(HY12), 2311(HY12), 2312(PO6), 2313(PO6), 2314(ST10), 2315(HY12), 2316(HY12), 2317(HY12), 2318(WW4), 2319(SM6), 2320(SM6), 2321(ST10), 2322(ST10), 2323(HW4)[c], 2324(CP2)[c], 2325(SM6)[c], 2326(WW4)[c], 2327(IR4)[c], 2328(PO6)[c], 2329(ST10)[c], 2330(CP2).

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JANUARY: 2331(EM1), 2332(EM1), 2333(EM1), 2334(EM1), 2335(HY1), 2336(HY1), 2337(EM1), [illegible], 2339(HY1), 2340(HY1), 2341(SA1), 2342(EM1), 2343(SA1), 2344(ST1), 2345(ST1), 2346(ST1), 2347(ST1), 2348(SM1)[c], 2349(HY1)[c], 2350(ST1), 2351(ST1), 2352(SA1)[c], 2353(ST1)[c], 2354(ST1).

FEBRUARY: 2355(CO1), 2356(CO1), 2357(CO1), 2358(CO1), 2359(CO1), 2360(CO1), 2361(PO1), 2362(HY2), 2363(ST2), 2364(HY2), 2365(PO1), 2366(HY2), 2367(PO1), 2368(SM1), 2369(HY2), 2370(SU1), 2371(HY2), 2372(PO1), 2373(SM1), 2374(HY2), 2375(PO1), 2376(HY2), 2377(ST2), 2378(SU1), 2379(SU1), 2380(SU1), 2381(ST2), 2382(ST2), 2383(SU1), 2384(ST2), 2385(SU1)[c], 2386(SU1), 2387(SU1), 2388(SU1), 2389(SM1), 2390(ST2)[c], 2391(SM1)[c], 2392(PO1)[c].

MARCH: 2393(IR1), 2394(IR1), 2395(IR1), 2396(IR1), 2397(IR1), 2398(IR1), 2399(IR1), 2400(IR1), 2401(IR1), 2402(IR1), 2403(IR1), 2404(IR1), 2405(IR1), 2406(IR1), 2407(SA2), 2408(SA2), 2409(HY3), 2410(ST3), 2411(SA2), 2412(HW1), 2413(WW1), 2414(WW1), 2415(HY3), 2416(HY3), 2417(HW1), 2418(HW1)[c], 2419(WW1)[c], 2420(WW1), 2421(WW1), 2422(WW1), 2423(WW1), 2424(SA2), 2425(SA2)[c], 2426(HY3)[c], 2427(ST3).

APRIL: 2428(ST4), 2429(HY4), 2430(PO2), 2431(SM2), 2432(PO2), 2433(ST4), 2434(EM2), 2435(ST4), 2436(ST4), 2437(ST4), 2438(HY4), 2439(EM2), 2440(EM2), 2441(ST4), 2442(EM2), 2443(HY4), 2444(ST4), 2445(EM2), 2446(ST4), 2447(EM2), 2448(EM2), 2449(HY4), 2450(ST4), 2451(ST4), 2452(HY4), 2453(SM2), 2454(SM2), 2455(EM2)[c], 2456(HY4)[c], 2457(PO2)[c], 2458(ST4)[c], 2459(SM2)[c].

MAY: 2460(AT1), 2461(ST5), 2462(AT1), 2463(AT1), 2464(CP1), 2465(CP1), 2466(AT1), 2467(AT1), [illegible], 2469(ST5), 2470(ST5), 2471(SA3), 2472(SA3), 2473(ST5), 2474(SA3), 2475(ST5), 2476(SA3), [illegible], (HY5), 2479(SA3), 2480(ST5), 2481(SA3), 2482(CO2), 2483(CO2), 2484(HY5), 2485(HY5), 2486(AT1)[c], 2487(CP1)[c], 2488(CO2)[c], 2489(ST5)[c], 2490(SA3)[c], 2491(ST5)[c], 2492(CP1), 2493(CO2).

JUNE: 2494(IR2), 2495(IR2), 2496(ST6), 2497(EM3), 2498(IR2), 2499(EM3), 2500(EM3), 2501(EM3), 2502(EM3), 2503(HY6), 2504(WW2), 2505(EM3), 2506(HY6), 2507(WW2), 2508(PO3), 2509(ST6), 2510(EM3), 2511(EM3), 2512(ST6), 2513(EM3), 2514(HY6), 2515(PO3), 2516(EM3), 2517(WW2), 2518(WW2), 2519(EM3), 2520(PO3), 2521(HY6), 2522(EM3), 2523(ST6), 2524(HY6), 2525(HY6), 2526(HY6), 2527(IR2), 2528(IR2)[c], 2529(IR2), 2530(IR2), 2531(HY6), 2532(EM3)[c], 2533(WW2)[c], 2534(WW2), 2535(ST6)[c], 2536(IR2)[c], 2537(PO3)[c], 2538(EM3)[c], 2539(IR2)[c], 2540(WW2)[c].

JULY: 2541(ST7), 2542(ST7), 2543(SA4), 2544(ST7), 2545(ST7), 2546(HY7), 2547(ST7), 2548(SU2), 2549(SA4), 2550(SA4), 2551(HY7), 2552(ST7), 2553(SU2), 2554(SA4), 2555(ST7), 2556(SA4), 2557(SA4), 2558(SA4), 2559(ST7), 2560(SU2)[c], 2561(SA4)[c], 2562(HY7)[c], 2563(ST7)[c].

c. Discussion of several papers, grouped by divisions.